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STOCHASTIC MODELING
FOR
AIRLIFT MOBILITY

by

David A. Goggins

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Thesis Advisor: David P. Morton

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This thesis is a continuation of optimization modeling research conducted at the Naval Postgraduate School for the U.S. Air Force Studies and Analyses Agency. That work resulted in Throughput II, a multi-period model for determining the maximum on-time throughput of cargo and passengers that can be transported with a given fleet over a given network, subject to appropriate physical and policy constraints. This and other existing deterministic strategic airlift models assume all data are known prior to making a decision; often times these assumptions are unrealistic. One such assumption is aircraft reliability. This thesis addresses the uncertainty of aircraft reliability, which, if ignored, can result in models that are overly optimistic with respect to throughput capability. To address this issue, this thesis adds a stochastic extension to Throughput II, resulting in a two-stage stochastic linear program with recourse that is solved using Benders decomposition. To analyze the stochastic program, a simulation model of the strategic airlift system is also developed. This simulation model allows the user to analyze the deterministic and stochastic models and to compare solutions. The stochastic model, in addition to the features of Throughput II, accomplishes the following: (1) selection of aircraft routes by anticipating potential bottlenecks in the system, (2) prevents unreliable aircraft from using capacity limited airfields and (3) a flow of cargo from origin to destination that is not interrupted by the random events of aircraft reliability.

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STOCHASTIC MODELING FOR AIRLIFT MOBILITY

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ABSTRACT

This thesis is a continuation of optimization modeling research conducted at the Naval Postgraduate School for the U.S. Air Force Studies and Analyses Agency. That work resulted in Throughput II, a multi-period model for determining the maximum on-time throughput of cargo and passengers that can be transported with a given fleet over a given network, subject to appropriate physical and policy constraints. This and other existing deterministic strategic airlift models assume all data are known prior to making a decision; often times these assumptions are unrealistic. One such assumption is aircraft reliability. This thesis addresses the uncertainty of aircraft reliability, which, if ignored, can result in models that are overly optimistic with respect to throughput capability. To address this issue, this thesis adds a stochastic extension to Throughput II, resulting in a two-stage stochastic linear program with recourse that is solved using Benders decomposition. To analyze the stochastic program, a simulation model of the strategic airlift system is also developed. This simulation model allows the user to analyze the deterministic and stochastic models and to compare solutions. The stochastic model, in addition to the features of Throughput II, accomplishes the following: (1) selection of aircraft routes by anticipating potential bottlenecks in the system, (2) prevents unreliable aircraft from using capacity limited airfields and (3) a flow of cargo from origin to destination that is not interrupted by the random events of aircraft reliability.

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EXECUTIVE SUMMARY

The magnitude of the airlift effort during Desert Shield/Storm was unprecedented. At the height of the war during Fall 1990, the Air Force averaged 17 million ston-miles per day of cargo and troops and by 10 March 1991, strategic airlift had moved more than 500,000 people and 540,000 stons of cargo (Gulf War Air Power Survey, 1993, p. 3). If the United States was to experience a future Operation Desert Shield/Storm type scenario, massive amounts of equipment and large numbers of personnel would have to be transported over continents and oceans, with an impending deadline. The magnitude of such a deployment imposes great strains on current and future air, land and sea mobility systems.

After the Gulf War in 1991, Congress commissioned a Mobility Requirements Study (MRS) of the United States Armed Forces. The MRS studied all aspects of mobility (domestic transportation, inter-theater lift, intra-theater lift and prepositioning) to determine the proper mix of sea, air, and amphibious lift, surface transportation and prepositioning. The goal was to provide Congress with an integrated plan for procuring the necessary lift for power projection in the 21st century. Two linear programming (LP) optimizations models were developed as part of the MRS. They are the Mobility Optimization Model (MOM) developed by the Joint Staff's Force Structure, Resource and Assessment Directorate (J8) (Wing *et al.*, 1991) and the Throughput model developed by the USAF/SAA (Yost, 1994).

In 1994 the Naval Postgraduate School conducted research in response to a request from the U.S. Air Force Studies and Analyses Agency (USAF/SAA) and culminated with the development of Throughput II, a model described in a 1994 NPS M.S. thesis by Capt. Lim Teo Weng (Lim, 1994) and enhanced the following year (Morton, *et al.*, 1995). Throughput II is a strategic airlift assets optimization problem formulated as a multi-period, multi-commodity flow model with a large number of side constraints. It is implemented in the General Algebraic Modeling System (GAMS) (Brooke *et al.*, 1992), and its purpose is to minimize late and non-deliveries subject to physical and policy constraints, such as aircraft utilization limitations and airfield handling capacities.

Throughput II and other existing strategic airlift optimization models typically assume all data is known prior to making a decision. However, assuming that all data is known with certainty is not always realistic. Specifically, existing optimization models fail to properly address aircraft reliability, which is an inherently random aspect of a strategic airlift system. Grounded aircraft that require repair work can significantly degrade the performance of an airlift system. Failing to properly model aircraft reliability may result in models that are too optimistic with respect to throughput capability. Another limitation is these optimization models may schedule unreliable aircraft through capacity limited airfields or airfields that have limited repair capability.

Techniques from stochastic programming are used to develop a model that incorporates aircraft reliability. Data is available for aircraft breakdown and repair rates, thus allowing the development of empirical probability distributions. As a result, aircraft reliability can be modeled as a random variable with a known distribution. With this data, a stochastic extension is added to the Throughput II model resulting in a two-stage stochastic linear program with recourse. The stochastic model, in addition to the features of Throughput II, accomplishes the following:

- Selection of aircraft routes by anticipating potential bottlenecks in the system.
- Minimizes the number of unreliable aircraft using capacity limited airfields or airfields that have limited repair capability.
- Achieve a flow of cargo to the theater that is not interrupted by the random events of aircraft reliability.

However, there is a price to be paid with respect to model size for incorporating aircraft reliability. Due to the large number of scenarios involved, this linear program is large in scale and beyond the capability of modeling languages like GAMS. To overcome this challenge, a special purpose algorithm designed for stochastic optimization models is utilized, specifically Benders decomposition.

To objectively analyze the deployment schedule recommended by the stochastic program, a detailed discrete-event simulation model of the strategic airlift system is also developed. The simulation model attempts to execute deployment schedules and allows the user to analyze the realism of the recommended schedule. Specifically, the simulation model is used to execute the deployment schedule from an optimization solution and allows the user both to analyze the deterministic and stochastic models and also to compare their proposed deployment schedules.

I. STOCHASTIC MODELING FOR AIRLIFT MOBILITY

Strategic airlift is a critical factor in military operations. The demand to distribute personnel, supplies, and equipment to combat areas can be immediate and of immense proportions. To project power across the world, strategic airlift, along with sealift, must transport the necessary units and equipment and also sustain those units. During Operation Desert Shield strategic airlift delivered 15 percent of the approximately 3.5 million stons of dry cargo (Gulf War Air Power Survey, 1993, p. 6), and between 7 August 1990 and 10 March 1991, airlift delivered approximately 23 percent of all cargo (Gulf War Air Power Survey, 1993, p. 112).

This thesis examines the major issue of airlift mobility and is a continuation of optimization modeling research conducted at the Naval Postgraduate School in 1994. In that year research was performed in response to a request from the U.S. Air Force Studies and Analyses Agency (USAF/SAA) and culminated with the development of Throughput II, a model described in a 1994 NPS M.S. thesis by Capt. Lim Teo Weng (Lim, 1994) and enhanced the following year (Morton, *et al.*, 1995). This thesis describes a stochastic extension to Throughput II to account for aircraft reliability. A detailed discrete-event simulation model of the strategic airlift system is also developed so that this stochastic system may be more closely analyzed. This simulation model attempts to execute deployment schedules and allows the user to analyze the deterministic and stochastic models and to compare various deployment schedules.

A. BACKGROUND

The magnitude of the airlift effort during Desert Shield/Storm was unprecedented. At the height of the war during Fall 1990, the Air Force averaged 17 million ston-miles per day of cargo and troops and by 10 March 1991, strategic airlift had moved more than 500,000 people and 540,000 stons of cargo (Gulf War Air Power Survey, 1993, p. 3). If the United States was to experience a future Operation Desert Shield/Storm type scenario, massive amounts of equipment and large numbers of personnel would have to be transported over continents and oceans, with an impending deadline. The magnitude of such a deployment imposes great strains on current and future air, land and sea mobility systems.

Despite the strategic success of Operation Desert Storm/Shield, the airlift operation experienced execution planning problems and a shortfall in airlift capability. Examples of each are as follows:

- Execution. Computer models could not analyze the airlift schedule and determine where the flow exceeded the throughput capacity of the airfield structure. Consequently, too many aircraft were passing through some parts of the system at one time, and airfields became backlogged. As a result the Air Force had to halt the flow on several occasions. (Gulf War Air Power Survey, 1993, p. 91)

- Shortfall. The Air Force relied heavily on the civil airline industry to fulfill its airlift requirements. Furthermore, without the thousands of missions flown by civilian air carriers, the Air Force could not have moved the required troops and cargo to the Arabian Peninsula by the time the United Nations deadline expired on 15 January 1991. (Gulf War Air Power Survey, 1993, p. 3)

The U.S. military services are currently working to improve the airlift system, and as a result, various optimization and simulation models have been developed. These analytical tools have been used to help improve the effectiveness of limited lift assets in order to minimize delivery shortfalls.

After the Gulf War in 1991, Congress commissioned a Mobility Requirements Study (MRS) of the United States Armed Forces. The MRS studied all aspects of mobility (domestic transportation, inter-theater lift, intra-theater lift and prepositioning) to determine the proper mix of sea, air, and amphibious lift, surface transportation and prepositioning. The goal was to provide Congress with an integrated plan for procuring the necessary lift for power projection in the 21st century. Two linear programming (LP) optimizations models were developed as part of the MRS. They are the Mobility Optimization Model (MOM) developed by the Joint Staff's Force Structure, Resource and Assessment Directorate (J8) (Wing *et al.*, 1991) and the Throughput model developed by the USAF/SAA (Yost, 1994).

1. Mobility Optimization Model

MOM is a multi-period, multi-commodity model with a single source in the United States and a single destination in the theater of concern. This model considers both air and sea mobility and is used in determining the proper level and mix of lift assets (air, sea and prepositioning) necessary to support U.S. power projection needs. Although MOM serves its purpose well by modeling the spectrum of strategic lift, it is not suitable for answering the detailed air mobility questions currently sought by USAF/SAA. In short, it does not model the airlift infrastructure network and constraints in sufficient detail; e.g., the maximum number of planes on the ground (MOG) constraint at airfields is not directly modeled.

2. Throughput Model

The Throughput model is a single-period, multi-commodity model which incorporates the strategic airlift network. This static airlift mobility model was developed by USAF/SAA to determine which part of the airlift system is restricting flow. It has also been used to evaluate alternative military strategic airlift fleets, route structures and basing schemes for different scenarios. Although the Throughput model has proven useful for USAF/SAA, it has inherent limitations due to its static nature (single period). For example, important constraints such as delivery time windows cannot be modeled when the time domain is not incorporated into the model. In addition, the model is unable to provide answers to key questions that concern decision-makers such as, on what day will Unit X commence movement and on what day will it be closed?

B. THROUGHPUT II

Throughput II is a strategic airlift assets optimization problem formulated as a multi-period, multi-commodity flow model with a large number of side constraints. It combines the time period aspect of MOM with the infrastructure network of Throughput. It is implemented in the General Algebraic Modeling System (GAMS) (Brooke *et al.*, 1992), and its purpose is to minimize late and non-deliveries subject to physical and policy constraints, such as aircraft utilization limitations and airfield handling capacities. For a given fleet and infrastructure, the model can provide insight when answering many mobility questions, such as: 1) Are the aircraft and airfield assets adequate for the deployment scenario? 2) What are the impacts of shortfalls in airlift capability? 3) Where are the system bottlenecks and when will they become noticeable? These types of analyses can be useful when selecting airlift assets and investing or divesting in airfield infrastructure. (Morton *et al.*, 1995)

In the Air Force analysis community, simulation modeling is currently used more widely than optimization. Simulation models more readily accommodate uncertainty and handle higher levels of detail, such as tracking individual airplanes and flight crews. For example, the Air Mobility Command's (AMC) Mobility Analysis Support System (MASS), a simulation model, is a discrete-event global airlift simulation of military and commercial airlift assets used in strategic and theater operations. MASS is the current standard for all airlift studies and in recent years, no critical airlift analysis has been performed without comparing the results with output from MASS. Yet even MASS has disadvantages; it can only answer "what-if" questions, not "what's best" questions; e.g., are there enough airlift assets for this scenario vs. which aircraft are the most efficient and versatile? Furthermore, the simulation program expends more manpower time and computer resources than optimization models. For instance, it requires two weeks to setup, run and analyze results for a typical MASS scenario.

C. PROBLEM STATEMENT

Throughput II and other existing strategic airlift optimization models typically assume all data is known prior to making a decision. However, assuming that all data is known with certainty is not always realistic. Specifically, existing optimization models fail to properly address aircraft reliability, which is an inherently random aspect of a strategic airlift system.[1] Grounded aircraft that require repair work can significantly degrade the performance of an airlift system. Failing to properly model aircraft reliability may result in models that are too optimistic with respect to throughput capability. Another limitation is these optimization models may schedule unreliable aircraft through capacity limited airfields or airfields that have limited repair capability.

D. METHODOLOGY

Techniques from stochastic programming are used to develop a model that incorporates aircraft reliability. Data is available for aircraft breakdown and repair rates, thus allowing the development of empirical probability distributions. As a result, aircraft reliability can be modeled as a random variable with a known distribution.

The stochastic optimization model is based on two-stage stochastic linear programming with recourse. The decision making process and uncertain events unfold in the following manner. First, an aircraft schedule is devised. Then, based on this schedule, the model examines all possible scenarios involving aircraft repair time realizations and the relation of these repair times to the available airfield capacities. If, for a specific scenario and time period the available capacity at an airfield is exceeded, a penalty in proportion to the excess will be incurred. The penalty function will discourage deployment schedules that frequently (as measured by the probability distribution) result in airfield capacity violations. This will produce an optimal solution that maximizes the expected value of system throughput–a flow of cargo from origin to destination that is not interrupted by the random events of aircraft reliability. This approach represents the logic of an experienced scheduler in that, relative to a deterministic model, the stochastic model is encouraged to:

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[1] While MASS has the capability of running with specified probability distributions for ground time, it is rarely run in this mode due to computational requirements (Waisanen, 1994).

- Select aircraft routes by anticipating potential bottlenecks in the system.
- Prevent unreliable aircraft from using capacity limited airfields or airfields that have limited repair capability.
- Achieve a flow of cargo to the theater that is not interrupted by the random events of aircraft reliability.

However, there is a price to be paid with respect to model size for incorporating aircraft reliability. Due to the large number of scenarios involved, this linear program is large in scale and beyond the capability of modeling languages like GAMS. To overcome this challenge, a special purpose algorithm designed for stochastic optimization models is utilized, specifically Benders decomposition.

To objectively analyze the deployment schedule recommended by the stochastic program, a detailed discrete-event simulation model of the strategic airlift system is also developed. The simulation model attempts to execute deployment schedules and allows the user to analyze the recommended schedule. Specifically, the simulation model is used to execute the deployment schedule from an optimization solution and allows the user both to analyze the realism of the deterministic and stochastic models and also to compare their proposed deployment schedules. Also, examining the results of an optimization model via a simulation model can provide useful feedback with respect to the validity of various linear programming modeling assumptions.

In Chapter II, the features, assumptions and limitations of the Throughput II model are discussed. The details of the mathematical formulation can be found in Appendix A. Also in Chapter II the issue of Maximum on Ground (MOG) and the impact of aircraft reliability on MOG is discussed. This is followed by the stochastic extension to Throughput II in Chapter III. The Benders decomposition algorithm used to solve the large stochastic model is described in Chapter IV. The simulation model used to compare the stochastic and deterministic models is developed in Chapter V and the analytical results and insight derived from this comparison are contained in Chapter VI.

II. DETERMINISTIC MODEL AND AIRCRAFT RELIABILITY

MOG (Maximum on Ground) is used extensively in the strategic airlift system and has a dramatic impact on throughput capability. Therefore, any detailed model of the airlift system must incorporate MOG. An explanation of MOG and why it is important is contained in Section A. Then in Section B, this thesis examines the model features and assumptions of an existing deterministic optimization model, Throughput II. As stated earlier, this deterministic model assumes perfectly reliable aircraft and deterministic ground times. But these assumptions err on the side of optimism. This shortcoming is also discussed in Section B. To examine the issue of aircraft reliability, Section C discusses the available reliability data and its impact on strategic airlift operations.

A. MAXIMUM ON GROUND (MOG) OR AIRFIELD CAPACITY

A key factor in any strategic airlift model is MOG. By definition MOG is "the highest number of aircraft being used in an operation which will be allowed on the ground during a given span of time based on simultaneous support." The key phrase in this definition is "simultaneous support;" an airfield's MOG value is the maximum number of aircraft that can be serviced at the same time when the airfield is equipped with specified service facilities. The MOG value for an airfield depends on a number of factors including parking spaces available (also called ramp space), material handling equipment, maintenance capabilities, fuel availability, as well as the type of service required (onload, refuel, or offload). Comparative scenarios are as follows:

- Scenario A: Airfield Bravo is being used as an enroute base. In this role the limiting factor for airfield Bravo is the number of fuel pumps available. Assuming the airfield can only refuel eight aircraft simultaneously, a MOG of eight is established.
- Scenario B: Airfield Bravo is an onload base. In this case the airfield may be limited by its ability to onload equipment due to a limited number of forklifts and/or cranes. Assuming the airfield can only load two aircraft simultaneously, this results in a MOG of two. Thus scenario B is more limiting than the fuel pump scenario of A.

As noted above MOG is not static; depending on the scenario and its limiting factors, MOG, for example, can range from a value of eight to a value of two.

A further variable related to MOG is ramp space. This is the maximum number of aircraft that the airfield can park at any given time. Thus for any airfield, its MOG value will be less than or equal to its ramp space. In an airlift operation, one might be inclined to send more planes above the MOG value because the large ramp space available at an airfield is believed to allow a larger flow of cargo to the theater. The following quote illustrates why MOG is important (versus ramp space) and why we should schedule in such a way to not violate MOG.

> "As more and more aircraft enter a MOG constrained system, it becomes progressively more difficult to serve these arriving aircraft in a scheduled ground time. The coordination of limited fueling, maintenance, loading and manpower assets to maximize throughput depends upon a limited number of aircraft per day. Adding aircraft can magnify the delays experienced for all users and throughput to a theater can and does decrease as the number of aircraft increases. Somalia is a good example. Early attempts to maximize flow rates into the country resulted in holding, diverts, malpositioned cargo, and rescheduled missions. All because too

> many aircraft were trying too operate in too constrained an environment. Ramp space is just the beginning. In Desert Shield, more aircraft should certainly deliver more to a huge ramp like the one at Dhahran: but what happened? Airlift aircraft waited behind commercial freighters, fighters, bombers, and other multinational forces waiting for limited supplies of fuel. Ground times increased to the point that we held missions in Europe for scheduled slot times down range. Slot times were not the result of inadequate ramp space, but were a solution to lengthy ground times waiting for fuel. These delays extended crew days beyond legal limits. It was not an effective strategy to send in a maximum flow. We achieved maximum throughput when scheduled slot times reduced the number of aircraft arriving per day." (Merril and Szabo, 1994, p. 4)

This quote indicates that the limiting factor in Dhahran for throughput capability is not ramp space but is fuel capacity (available to airlift aircraft), which defines its MOG value.

As stated earlier MOG is the maximum aircraft on the ground an airfield can service simultaneously. But in airlift operations and in the modeling world, it is convenient at times to apply a transformation to the definition of MOG; MOG can be transformed to the maximum flow rate of inbound aircraft an airfield can sustain. For instance, if an airfield has a MOG value of two (assuming one hour ground times), this airfield can support aircraft arriving every 30 minutes. In military airlift operations, this transformation of MOG enables personnel in the Air Force to make better scheduling decisions. This transformation is also necessary for models with discrete time periods–in particular the mathematical optimization models discussed and developed in this thesis.

Using the transformation of MOG, is it realistic to have aircraft arrive at this maximum flow? This question highlights two separate issues.

1. Airlift operations. The current planning factors require an aircraft be serviced in an allotted amount of time for refueling, offloading or onloading. But what

happens when an aircraft breaks down? To remedy the situation, additional manpower or equipment resources will be required. But from where? Since the airfield has no additional resources readily available, the only solution is to divert resources from other aircraft being serviced, thus multiplying the problem situation and decreasing capacity even further. And as more aircraft arrive, significant delays can develop while the base is operating beyond its current capacity.

2. Modeling. In building a mathematical model with discrete time steps, MOG is transformed from a physical capacity (i.e., 2 plane slots) into a unit that involves time (i.e., 48 plane slots per time period; assuming a 24 hour time period and one hour ground times). From the perspective of the model, it is perfectly acceptable to have all aircraft (i.e., 48 aircraft) arrive at one time. However, to fully utilize this capacity in reality, the model must produce a perfect back-to-back schedule (i.e., aircraft must arrive every 30 minutes). If aircraft arrivals are not uniformly spaced, even if aircraft do not break down, the model cannot achieve this capacity in reality, even though the model thinks it can.

To remedy this potential problem, one should lower the arrival rate or flow of aircraft to this airfield from the maximum. This will result in 1) more manpower and equipment immediately available for broken aircraft and 2) minimize the possible violation of MOG in the optimization model due to discrete time periods. This will allow aircraft to land and immediately be serviced, thus achieving a realistic flow of cargo from origin to destination. This thesis will determine, by modeling one major aspect of ground time, namely aircraft reliability, the proper rate of flow of aircraft from origin to destination. This flow rate can then be transformed to determine at what percentage of MOG an airfield should operate.

B. THROUGHPUT II

The original Throughput II model was developed by Capt. Lim Teo Weng (Lim, 1994). Since then the model has been enhanced in ongoing research at the Naval Postgraduate School (Morton, *et al.*, 1995). A brief overview of the mathematical formulation of Throughput II is in Appendix A.

1. Model Features

The Throughput II model is a multi-period model for determining the maximum on-time throughput of cargo and passengers that can be transported with a given fleet over a given network, subject to appropriate physical and policy constraints. The objective function minimizes the total weighted penalties for late deliveries and non-deliveries. This model considers only inter-theater, not intra-theater deliveries. The major features of the airlift system currently captured by the model include (Morton, *et al,* 1995):

- Multiple origins and destinations. The model flies aircraft through multiple origin, enroute and destination airfields. Due to the size of the airlift system, aggregation of origin and destination airfields is done to allow tractability of the model.

- Flexible routing structure. The air route structure supported by the model includes delivery and recovery routes with a variable number of enroute stops (usually between zero and three). This will give the model the option of short-range flights with heavier loads or long-range flights with lighter loads. For further routing flexibility, the model also allows the same aircraft to fly different delivery and recovery routes on the same mission.

- Aircraft-to-route restrictions. The user may impose aircraft-to-route restrictions; e.g., military aircraft may only use military airfields for enroute stops. This particular provision arises because the USAF AMC may call upon civilian commercial airliners to augment USAF aircraft in a deployment, under the Civil Reserve Airfleet (CRAF) program. The model distinguishes between USAF and CRAF aircraft.

- Aircraft assets can be added over time. This adds realism to the model, because CRAF and other aircraft may take time to mobilize and are typically unavailable at the start of a deployment.
- Delivery time windows. In a deployment, a unit is ready to move on its available-to-load-date (ALD) and has to arrive in the theater by its required-delivery-date (RDD). This aspect of the problem has been incorporated in the model through time windows that are specified for each unit. The model treats the time window as "elastic" in that the cargo may be delivered late, subject to a penalty.

2. Assumptions

The assumptions used in the model are as follows (Morton, *et al.*, 1995):

- Congestion. Inventoried aircraft at origin and destination airfields are considered not to affect the aircraft handling capacity of the airfield. This assumption is not strictly valid since an inventoried aircraft may interfere with the ability of the airfield to process aircraft in the airlift operation.
- Deterministic ground time. Aircraft turnaround times for onloading and offloading cargo and enroute refueling are assumed to be known constants, although they are naturally stochastic. This ignores the fact that deviations from the given service time can cause congestion on the ground. To offset the optimism of this assumption, Throughput II uses an efficiency factor (<1) in the formulation of MOG consumption constraints to lessen the impact of randomness.
- Computational tractability. The model aggregates onload and offload airfields, and uses continuous decision variables and discrete time periods–typically one day.

3. Limitations

Throughput II errs on the side of optimism with respect to throughput capability due to the assumption of deterministic ground times and perfectly reliable aircraft. Also the single-day time periods can decrease the model's fidelity. This results in the following potential problems that will be examined in this thesis:

- Deterministic ground times for aircraft. The model, if needed, will continually saturate airfield capacities to achieve the maximum flow of cargo from origin to destination. However, the aircraft ground times used in the calculation of MOG consumption represent the expected times for onload/refuel/offload, resulting in optimistic throughput capability (via Jensen's inequality). In an attempt to overcome this optimism, the deterministic model uses a MOG efficiency factor to lessen the impact of random ground times.

- Perfectly reliable aircraft. Broken aircraft can have an immediate impact on throughput capability, especially when airfield capacity resources are limited. Moreover, the necessary repair work consumes resources that might be needed elsewhere. The location of the broken aircraft is also critical. If an aircraft breaks enroute, not only does the necessary repair work consume resources at the enroute airfield, the cargo onboard the aircraft will be delayed in arriving at its destination. If the aircraft breaks at an airfield with limited maintenance capability, the required maintenance facilities and assets might not be readily available, resulting in even longer delays.

- Time resolution. With one-day time periods, the model can route aircraft in a manner that causes local congestion. For example, all aircraft could arrive at an airfield within a small time window instead of being dispersed over an entire day. In reality this would cause local congestion, even though the model's representation of aircraft handling capacity is observed. Another limitation is that the model rounds the time (to the nearest day) at which an aircraft arrives at origin and destination airfields.

C. AIRCRAFT RELIABILITY

There are many factors that contribute to the stochastic nature of ground times for aircraft. Some of these factors include local congestion at an airfield, lack of maintenance personnel or spare parts, airlift users, aircraft reliability, and weather. Aircraft reliability accounts for a significant portion of random ground times.

1. Data

The Air Mobility Command (AMC) collects aircraft breakdown and repair rates for each of the strategic military aircraft, which are defined as follows (AMC Pamphlet 21-2, p. 17):

- Break: system malfunction that renders aircraft non-mission capable (NMC) after landing.
- Break rate: percent of aircraft landings that have system malfunctions rendering aircraft NMC.
- Break rate formula:

$$\left(\frac{\text{number of breaks}}{\text{number of landings}}\right) * 100. \tag{2.1}$$

- Fix: completing maintenance actions on NMC aircraft rendering the aircraft airworthy (PMC, partially mission capable, or FMC, fully mission capable).
- Fix rate: percent of aircraft landing NMC that are fixed within established time frames (0-4, 4-8, 8-12, 12-16, 16-24, 24-48, 48-72, >72 hours).
- Fix rate formula:

$$\left(\frac{\text{number of aircraft fixed within a specified time frame}}{\text{number of aircraft landing breaks}}\right)*100. \qquad (2.2)$$

Appendix B contains data in this form from 1994 for the C5, C17 and C141. Based on this data, an empirical probability mass function, $p_a(t)$, for each aircraft type a is developed. Each possible realization of the random variable T_a, representing the required repair time, will have an associated probability, $P(T_a = t) = p_a(t)$. This probability mass function applies each time a plane lands to determine the required repair time, T_a. This assumes that the probability mass function does not depend on the hours flown by the aircraft (This assumption is not entirely correct, since aircraft reliability typically decreases with increasing flying hours.). With this assumption, the probability mass functions are derived resulting in nine realizations per aircraft (Table II-1).

Repair Time (hrs)	$p_{C5}(t)$	$p_{C17}(t)$	$p_{C141}(t)$
0	0.86540	0.93110	0.85110
2	0.03078	0.02733	0.06051
6	0.02933	0.00832	0.03379
10	0.01643	0.00238	0.02116
14	0.01582	0.00238	0.01205
20	0.01269	0.00832	0.00910
36	0.01872	0.01069	0.00713
60	0.00396	0.00712	0.00213
72	0.00686	0.00237	0.00304

Table II-1: Probability Mass Function, $p_a(t)$.

In review, we have the following notation:

- T_a - random variable representing the repair time of an aircraft of type a.
- $p_a(t)$ - the probability mass function for T_a.
- S_a - support of the random variable T_a, where $S_a = \{0, 2, 6, 10, 14, 20, 36, 60, 72\}$.

To consider aircraft reliability, a model can be formulated that tracks each individual aircraft. By using binary variables, the model can determine if that specific aircraft breaks when it lands at airfield b in time period t, thus the binary variable will require three indices (aircraft identification number, airfield and time period). But because the model does not know a priori how to schedule the aircraft, the model does not know which airfield the plane will land at on any given time period. Therefore the model must consider every possible scenario that can occur. For a singe aircraft, the size of the sample space (assuming nine realizations per aircraft) is

$$|\Omega| = 9^{\text{(total number of airfields)*(time periods)}}. \tag{2.3}$$

In extending this to all aircraft in the model, the size of the sample space (assuming 9 realizations for each aircraft) is

$$|\Omega| = 9^{\text{(total number of aircraft)*(total number of airfields)*(time periods)}}. \tag{2.4}$$

To examine these scenarios in a mathematical program, the model will require a constraint for each scenario (as a minimum); and a binary variable for each individual aircraft (as an indicator variable) to determine if that particular plane lands at airfield b in time period t. Thus the size of the model is proportional to the size of the sample space. Using the Throughput II data considered by Lim (1994), this results in a minimum of

$$|\Omega| = 9^{(416\text{ aircraft})*(16\text{ airfields})*(30\text{ time periods})} = 9^{199,680} \tag{2.5}$$

constraints, in addition to the requirement for binary variables. A model involving this minimum number of constraints is impossible to solve given today's technology.

2. Scenario Development

To reduce the size of the sample space and the associated model, three steps are enacted. First, the convolution is used. For instance, the convolution of $p_a(t)$ with itself is

$$p_{a,2}(t) = \sum_{y \in S_a} p_a(t-y)p_a(y). \tag{2.6}$$

The convolution of $p_a(t)$ with itself n times is denoted $p_{an}(t)$: the probability mass function of the total repair time random variable, T_{an}, of n aircraft of type a. In the model we develop, the convolution procedure will be utilized to reduce the dimension of the model. In review, we have the following additional notation:

- T_{an} - random variable formed by summing n independent and identically distributed random variables, T_a, each with probability mass function $p_a(t)$.
- $p_{an}(t)$ - the probability mass function for T_{an} with support S_{an}.

To reduce the number of scenarios even further, the second step approximates the distribution of the random variable, T_{an} with another random variable, $\hat{T}_{an}$. This new random variable consists of a smaller number of realizations. While $\hat{T}_{an}$ has fewer realizations than T_{an}, the probability mass function $\hat{p}_{an}(t)$ still closely resembles that of $p_{an}(t)$. This results in the following notation:

- $\hat{T}_{an}$ - random variable that approximates T_{an} but has fewer realizations.
- $\hat{p}_{an}(t)$ - the probability mass function for $\hat{T}_{an}$ with support $\hat{S}_{an}$.

The third step incorporates the random variable $\hat{T}_{an}$ in the stochastic model developed in this thesis. We define the random variable

$$RTime_a = \frac{1}{n}\hat{T}_{an}; \qquad (2.7)$$

realizations of this random variable are denoted $RTime_a^{\omega}$. Multiplication of this random variable by the number of landings of aircraft of type a gives the total repair time required; this has the correct distribution if exactly n aircraft have landed. It will be shown in Chapter III that the stochastic model developed examines separately the possible scenarios for each base and time period, i.e., each (b,t) combination will have its own random variable $RTime_a$. Thus the correct notation is $RTime_{abt}$. In review we have the following notation:

- $RTime_{abt}$ - the repair time random variable for aircraft of type a at airfield b and time period t.
- $\hat{p}_{abt}(t)$ - the probability mass function for $RTime_{abt}$ with support $\hat{S}_{abt}$.

These three steps reduce the size of the sample space for each base and time period combination, Ω_{bt}, to (assuming nine scenarios per aircraft type ($n = 9$) for $\hat{p}_{an}(t)$)

$$|\Omega_{bt}| = 9^{(3 \text{ aircraft types})}. \tag{2.8}$$

This results in a minimum of an additional

$$9^{(3 \text{ aircraft types})} * (16 \text{ airfields}) * (30 \text{ time periods}) = 349{,}920 \tag{2.9}$$

constraints. The stochastic model developed in Chapter III will then require a minimum of 349,920 constraints. The advantages and disadvantages of these three steps are as follows:

Advantages:

- Reduces the number of scenarios or the size of the sample space.

- No need for integer programming. Since $RTime_{abt}$ approximates the repair time variable for a single aircraft of type *a*, at airfield *b*, time period *t*, multiplication of $RTime^{\omega}_{abt}$ by a continuous variable gives the approximate total repair time required in scenario ω (assuming the effects of using a continuous decision variable are negligible).

- A stochastic model using this sample space is too large to be solved using modeling languages like GAMS, but it can be solved using decomposition based algorithms.

Disadvantages:

- Since the input to the stochastic model will be $RTime_{abt}$, the realizations of this random variable will be exact if the model sends exactly *n* planes of type *a* to airfield *b* in time period *t*. In this case, the only error is caused by using $\hat{T}_{an}$ in approximating T_{an}. This error will get smaller as the number of realizations per aircraft type used in developing $\hat{T}_{an}$ approaches the number of realizations of T_{an}.

- If the model sends more or less than *n* planes, the realizations of $\hat{T}_{an}$ is not accurate since this was based on *n* planes. This error is in addition to the error caused by using $\hat{T}_{an}$ to approximate T_{an}.

D. EFFECT OF AIRCRAFT RELIABILITY ON AIRLIFT OPERATIONS

In developing the stochastic model, two separate modeling issues were of concern. First, a broken aircraft consumes a portion of ramp space and second, a broken aircraft will have some type of impact on MOG. Consider the following points relating to the issue of broken aircraft consuming ramp space:

- In the literature ramp space is never mentioned as the limiting factor for throughput capability. During the Gulf War, Rhein-Main had sixty-eight aircraft on the ramp, even though the base had parking spaces for only fifty-six aircraft (Gulf War Air Power Survey, 1993, p. 100).

- Ramp space data is not currently available to support a stochastic model with this model enhancement.

In light of these two points, this thesis does not attempt to model the impact of broken aircraft on ramp space, i.e., the stochastic model does not have a ramp space consumption constraint. On the other hand, consider the following points:

- A broken aircraft does consume ramp space, but the plane also requires resources to complete repairs–resources that could be used for processing planes in the delivery of cargo.

- In peacetime there is no urgent need to repair aircraft. But during a critical airlift operation, a broken plane has an immediate impact when aircraft supply is a limiting factor, the aircraft is carrying a critical load or the aircraft breaks at an airfield with limited maintenance capability. In these cases additional resources may be required to repair the plane.

- In an airlift operation the goal is to achieve the highest throughput capability possible, but this is directly related to MOG and not ramp space.

Thus, the stochastic model must mathematically model the impact of aircraft reliability on MOG.

Using another transformation of MOG, this thesis will be referring to hours of MOG consumed. For instance, if an airfield has a MOG of two, then this airfield has 48 hours of MOG available (24 hours per day * 2) per day. So when a plane does break, how many hours of MOG does the plane actually consume? In addition to the resources required for the onload/refuel/offload, an additional amount of resources is required to repair the aircraft. Converting repair times directly into MOG consumption is often too pessimistic; instead only a fraction of the total repair time actually reduces capacity. The longer the aircraft is on the ground the smaller that fraction. If the repair time random variable realization is two hours, for example, we assume that all repairs will consume an additional two hours of MOG. On the other hand, if the repair time random variable realization is 72 hours, we assume the following:

- If a plane breaks at a large enroute airfield with unlimited maintenance personnel, an additional six hours of MOG are consumed. This assumes six hours are required to offload and then in turn onload the cargo onto another plane, while the original plane is repaired at the hangar.
- If the plane broke at an airfield with very limited maintenance personnel, the plane will consume seventy-two hours of MOG.

Using these ideas, the following is performed to produce data used in the stochastic optimization model for each base and time period combination:

1. Obtain the probability mass function, p_a(t), for the random variable T_a for each aircraft of type *a*.
2. Modify the realizations of T_a to account for the amount of MOG hours consumed. This can depend upon the airfield *b* and the time period *t*.
3. Perform the convolution and approximation to obtain the random variable $\hat{T}_{ant}$.
4. Divide the realizations of $\hat{T}_{an}$ by *n* to obtain the random variable $MRTime_{abt}$.

III. STOCHASTIC OPTIMIZATION MODEL

Due to the variety of stochastic optimization models discussed in the literature, Section A gives a broad overview of stochastic programming terminology related to the stochastic optimization model developed in this thesis. Then in Section B, the stochastic extension to Throughput II is described, resulting in a two-stage stochastic linear program with recourse. This will permit a study of how the optimal solution changes when going from a deterministic model to a stochastic model that incorporates aircraft reliability (Chapter VI). It is shown in Section C that this stochastic optimization model is very large and must be solved using a decomposition based algorithm. The chapter then concludes with a summary of the underlying constructs used to develop the stochastic model and the additional model features gained (in addition to those of the Throughput II model) by using this stochastic extension.

A. STOCHASTIC PROGRAMMING

In deterministic linear programs (LP), all parameters are treated as known constants. This can lead to an optimistic or pessimistic objective function value, depending on which parameters are random. Moreover, in general it may lead to sub-optimal decisions because they fail to hedge against the full range of possible scenarios. On the other hand, stochastic programming models treat a subset of the parameters as random variables with a known distribution. Even when the "exact" distribution of these random variables can not be obtained, the stochastic model will be an improvement over the one-point forecast of the deterministic LP.

In the stochastic programming models presented here, realizations of random parameters are denoted with superscript ω, the corresponding sample space by Ω, and the probability mass function by p^{ω}. There are a number of options available in extending the following linear program

$$\begin{aligned} &\text{minimize} && cx \\ &\text{subject to} && Ax \geq b \\ &&& x \geq 0 \end{aligned} \tag{3.1}$$

to the stochastic domain. One such option is to insist that a feasible decision be made no matter what scenario might unfold:

$$\begin{aligned} &\text{minimize} && cx \\ &\text{subject to} && A^{\omega} x \geq b^{\omega} \qquad \forall \omega \in \Omega \\ &&& x \geq 0. \end{aligned} \tag{3.2}$$

Here we have assumed that the objective function coefficients, c, are known with certainty. Many decision makers regard (3.2) as too pessimistic a formulation because it is typically driven by the most "extreme" scenario. In most real-world systems, many of the structural constraints are elastic and may be violated at a certain cost. There are a variety of stochastic programming models that differ primarily in how this issue of "infeasibility" is handled. Those relating to the stochastic optimization model developed in this thesis are described below (Morton, 1994).

1. Simple Recourse Models

In this type of model, infeasibility is accepted, but only at a certain cost; with each constraint violation, there is an associated cost. Let $q = (q_1, q_2, q_3, \ldots, q_m)$ denote the unit cost of violating each constraint. The positive part of a scalar variable, s, is defined as $s^+ = \max\{s, 0\}$; the positive-part-operator may be applied to a vector, componentwise. With this definition, the simple recourse model is

$$\begin{aligned} &\text{minimize}_{x \geq 0} && cx + E_\omega h(x,\omega) && (3.3) \\ &\text{where} && h(x,\omega) = q(b^\omega - A^\omega x)^+, \end{aligned}$$

where $E_\omega h(x,\omega) = \sum_{\omega \in \Omega} p^\omega h(x,\omega)$.

2. Recourse Models

In recourse models infeasibility is corrected, afterwards, at a certain cost. The costs due to the amount of violation in the constraints are determined after the observation of the random data and are denoted as recourse costs; the expected value criterion is used. In this case the model is written as

$$\begin{array}{llll} \text{minimize} & cx + E_\omega h(x,\omega) & & \\ & x \geq 0 & & \\ \text{where} & h(x,\omega) = & \text{minimize} & fy \\ & & \text{subject to} & Wy \geq b^\omega - A^\omega x \\ & & & y \geq 0. \end{array} \tag{3.4}$$

As in the simple recourse case, $E_\omega h(x,\omega)$ is called the recourse function; W is called the recourse matrix. The recourse variable, y, measures the corresponding violation in the constraints, if any. If $W = I$, we have the simple recourse model.

The "standard" two-stage recourse model replaces the first stage constraints $x \geq 0$ with general structural inequalities and otherwise generalizes the problem as follows:

$$\underset{\substack{Ax=b \\ x\geq 0}}{\text{minimize}} \quad \{cx + E_\omega h(x,\omega)\}. \tag{3.5}$$

The second-stage cost we will incur, if we observe scenario ω and act optimally given first-stage decision x, is $h(x,\omega)$ and may be expressed as

$$h(x,\omega) = \text{minimize} \quad f^{\omega} y \tag{3.6}$$

$$\text{subject to} \quad W^{\omega} y = d^{\omega} + B^{\omega} x$$

$$y \geq 0.$$

The recourse aspect of the model gives it a unique characteristic. The model is two-stage: first the decision variables are chosen, then the stochastic variables are observed. These two steps will determine the recourse variables (to recover feasibility) and their corresponding penalties. These penalties depend on both the constraint violated and the magnitude of violation. The recourse cost has the effect of restraining "aggressive" choices of decision variables if the costs involved with regaining feasibility outweigh the benefits. Thus, the two-stage recourse model minimizes the sum of our original first-stage costs and the expected recourse costs.

B. MODEL

As stated in Chapter II, the deterministic Throughput II model minimizes late and undelivered cargo subject to system constraints. One set of these constraints, MOG consumption (see Appendix A, constraint A.15), contain inherently random coefficients because the ground times are of a random nature. This is where stochastic programming can be used. If the pessimistic formulation (3.2) is used, it would be driven by the largest conceivable ground time; that way we could ensure that we will never exceed MOG. However, this is an extreme and unacceptable solution because the airlift system would be dramatically underutilized. Thus instead we will permit certain scenarios to violate the MOG constraint, but add an objective function formulation that discourages the model from generating schedules that yield frequent or large violations (3.6).

In this section we describe a stochastic extension to Throughput II to include aircraft reliability. In the following pages, the first-stage variable x is a large vector that includes all decision variables of Throughput II (i.e., X_{uart}, Y_{art}, etc.). The following is added to the Throughput II model, resulting in a two-stage stochastic linear program model with recourse.

1. Additional Data

$MOGPEN_{bt}$	This penalty is used in the objective function when airfield (base) b, in time period t, exceeds MOG, referred as $MOGCap_b$; these correspond to the objective coefficients, f, in (3.6).
$MRTime^{\omega}_{abt}$	The modified repair time random variable for aircraft type a, at airfield b, at time period t, is denoted $MRTime_{abt}$. Realizations of this random variable are denoted $MRTime^{\omega}_{abt}$. These values correspond to coefficients in the B^{ω} matrix in (3.6).

2. Additional Decision Variable

R^{ω}_{bt}	The amount by which $MOGCap_b$ is exceeded at airfield b in time period t for scenario ω; this variable corresponds to the y variable in (3.6).

3. Additional Objective Function Term

The sample space for the vector of the modified repair times for all aircraft types at airfield b at time period t is denoted $\Omega_{bt} = \{1, 2, ..., K_{bt}\}$. From that sample space, each outcome $\omega(b,t) \in \Omega_{bt}$ has an associated probability $p_{bt}^{\omega(b,t)}$. This results in a recourse function for each (b,t) combination with the following notation

$$E_{\omega(b,t)} h_{bt}\left(x, \omega(b,t)\right) = \sum_{\omega(b,t) \in \Omega_{bt}} p_{bt}^{\omega(b,t)} h_{bt}(x, \omega(b,t)), \tag{3.7}$$

where

$$h_{bt}(x, \omega(b,t)) = MOGPEN_{bt} * R_{bt}^{\omega}. \tag{3.8}$$

In the following pages this thesis abbreviates *ω(b,t)* by *ω*, resulting in the following

$$E_{\omega} h_{bt}\left(x, \omega\right) = \sum_{\omega \in \Omega_{bt}} p_{bt}^{\omega} h_{bt}(x, \omega). \tag{3.9}$$

Using this notation, the following term is added to the original objective function of Throughput II:

$$\sum_{b} \sum_{t} E_{\omega} h_{bt}(x, \omega) = \sum_{b} \sum_{t} \sum_{\omega \in \Omega_{bt}} p_{bt}^{\omega} \left(MOGPEN_{bt} * R_{bt}^{\omega} \right). \tag{3.10}$$

This will penalize the model whenever the $MOGCap_b$ at airfield b, in time period t, scenario ω is exceeded. How the value of $MOGPEN_{bt}$ is determined and its impact on the model is discussed in Section D of this chapter.

4. Modification to Throughput II Constraints

Throughput II uses the deterministic ground time for calculating the time required for an aircraft to execute a route and for the amount of MOG consumed. For example, to calculate the time a loaded aircraft will arrive at the second enroute airfield, the model sums the onload ground time, one enroute ground time and the required flight time. Using the deterministic ground time is an optimistic assumption. Aircraft reliability, only one aspect of the ground time variability, can cause delays in the delivery of cargo. Since it is unreasonable (from a tractability perspective) to have aircraft actually "break" in a linear program, the model needs to indirectly model this issue. This can be achieved, in part, by using the expected value of the modified repair time, in addition to the deterministic ground time, for the calculation of MOG consumption and the time for an aircraft to execute a route. Even though the stochastic model uses the expected value of the modified repair time in the MOG consumption constraint, the optimal solution is optimistic (via Jensen's inequality). This optimistic solution is "modified" by the addition of new constraints.

5. Additional Constraints

In calculating the MOG consumption at airfield b in time period t, the Throughput II model uses deterministic ground times (see Appendix A, constraint A.15). As stated earlier, this will lead to an optimistic solution. To account for a portion of the random ground times, aircraft reliability is incorporated into this constraint by using the random variable $MRTime_{abt}$. The original constraints are maintained (for reasons to be discussed in Chapter IV) and the following constraints are added:

$$(1) \quad \sum_{u} \sum_{a} \sum_{r \in R_a} \sum_{\substack{t' \in T_{uar} \\ t' + \lfloor DTime_{abr} \rfloor = t}} \left[MOG\,\mathrm{Re}q_{ab} * \left(GTime_{abr} + MRTime_{abt}^{\omega} \right) / 24 \right] * X_{uart'} \qquad (3.11)$$

$$+ \sum_{a} \sum_{r \in R_a} \sum_{\substack{t' \in T_{uar} \\ t' + \lfloor DTime_{abr} \rfloor = t}} \left[MOG\,\mathrm{Re}q_{ab} * \left(GTime_{abr} + MRTime_{abt}^{\omega} \right) / 24 \right] * Y_{art'}$$

$$- R_{bt}^{\omega} \leq MOGCap_b \qquad \forall\, b,t,\omega$$

$$(2) \quad R_{bt}^{\omega} \geq 0 \qquad \forall\, b,t,\omega. \qquad (3.12)$$

The stochastic model will examine each scenario ω, at airfield b, time period t, and determine if the $MOGCap_b$ has been exceeded. If it has, R_{bt}^{ω} will be the amount of the violation.

C. AN EQUIVALENT REPRESENTATION OF THE MODEL

The deterministic formulation of Throughput II is a linear program and hence can be written in the following standard form:

$$\begin{aligned} \text{minimize} \quad & cx \\ \text{subject to} \quad & Ax = b \\ & x \geq 0. \end{aligned} \tag{3.13}$$

The stochastic extension to Throughput II results in the following model:

$$\begin{aligned} \text{minimize} \quad & cx + \sum_b \sum_t E_\omega h_{bt}(x,\omega) \\ \text{subject to} \quad & Ax = b \\ & x \geq 0, \end{aligned} \tag{3.14}$$

where $h_{bt}(x,\omega) = \underset{R_{bt}^{\omega}}{\text{minimize}} \; MOGPEN_{bt} * R_{bt}^{\omega}$

subject to

$$(1)\ \sum_{u}\sum_{a}\sum_{r\in R_a}\sum_{\substack{t'\in T_{uar}\\ t'+\lceil DTime_{abr}\rceil=t}}\left[MOG\,Req_{ab}*\left(GTime_{abr}+MRTime^{\omega}_{abt}\right)/24\right]*X_{uart'}$$

$$+\sum_{a}\sum_{r}\sum_{\substack{t'\in T_{uar}\\ t'+\lceil DTime_{abr}\rceil=t}}\left[MOG\,Req_{ab}*\left(GTime_{abr}+MRTime^{\omega}_{abt}\right)/24\right]*Y_{art'}$$

$$-R^{\omega}_{bt}\le MOGCap_b$$

$$(2)\ R^{\omega}_{bt}\ge 0.$$

Upon close examination of the stochastic extension to Throughput II, the additional constraints only examine the effect of the modified repair time random variable at airfield *b* and time period *t*. Thus the sample space for all aircraft at airfield *b* in time period *t*, Ω_{bt}, has a size of (see Chapter II, Section C)

$$\left|\Omega_{bt}\right| = 9^{(3\text{ aircraft types})} = 9^3 = 729, \tag{3.15}$$

resulting in an additional

$$729*(16\text{ airfields})*(30\text{ time periods}) = 349{,}920 \tag{3.16}$$

constraints. Combined with the Throughput II data considered by Lim (1994), we have a total of

$$(349{,}920 + 7{,}671) = 357{,}591 \tag{3.17}$$

constraints. The stochastic model also has 362,166 decision variables. This results in a very large model which cannot be solved using standard modeling languages like GAMS. However, it has a special structure that can be exploited by decomposition based algorithms, specifically Benders decomposition (Chapter IV). Using this algorithm, the stochastic optimization model is solved under 20 minutes on a IBM RS6000 model 590 workstation (the deterministic model, using GAMS, solves under two minutes), obtaining a near-optimal solution.

With the solution from the stochastic optimization model, this thesis examines the impact of aircraft reliability on strategic airlift. Specifically, the thesis examines the change in the optimal solution when going from a deterministic model to a stochastic model that incorporates aircraft reliability. The analysis is contained in Chapter VI.

D. CALCULATION OF PENALTY

The number used for $MOGPEN_{bt}$ can have a dramatic impact on the optimal solution. If $MOGPEN_{bt} = 0$ (for all airfields b and time periods t), the stochastic model will give the same solution as Throughput II (ignoring the use of $E[MRTime_{abt}]$). At the other extreme, setting $MOGPEN_{bt} = +\infty$ (for all airfields b and time periods t) will not allow the model to exceed $MOGCap_b$ in any scenario; i.e., we recover the pessimistic model (3.2). For example, if the worst case scenario had all aircraft break for 22 hours (assuming a 2 hour onload per aircraft and $MOGCap_b = 2$), then the stochastic model will only send two aircraft to the airfield. This illustrates that the value used for $MOGPEN_{bt}$ has a dramatic impact on the optimal solution.

As stated in Chapter II, one goal of the stochastic model is to determine the optimal rate of flow of aircraft from origin to destination, a flow of aircraft that is not interrupted by the random events of aircraft reliability. If the stochastic model exceeds MOG for a specific scenario, the model needs to incur a cost for attempting to use more resources than available. And this cost needs to be large enough so that the model chooses the penalty for not delivering the cargo versus the penalty of exceeding MOG. Specifically, when the model sends aircraft *a* to airfield *b*, this aircraft will require the following amount of resources to service the aircraft (assuming the aircraft does not break):

$$\text{resources needed} = MOG\,\mathrm{Re}q_{ab} * GTime_{abr}. \tag{3.18}$$

Now if airfield *b* has no additional resources available for this aircraft (i.e. it is operating at or above $MOGCap_b$), then the recourse variable R_{bt}^{ω} will increase by

$$R_{bt}^{\omega} = MOG\,\mathrm{Re}q_{ab} * GTime_{abr}. \tag{3.19}$$

The objective function will now increase by

$$MOGPEN_{bt} * R_{bt}^{\omega} = MOGPEN_{bt} * MOG\,\mathrm{Re}q_{ab} * GTime_{abr} . \qquad (3.20)$$

This increase in the objective function must be larger than the maximum penalty that will be incurred for not delivering the cargo,

$$MOGPEN_{bt} * MOG\,\mathrm{Re}q_{ab} * GTime_{abr} > \begin{bmatrix} \text{Maximum penalty incurred for not} \\ \text{delivering one plane - load of cargo} \end{bmatrix} . \qquad (3.21)$$

The above equation can now be solved for $MOGPEN_{bt}$.

E. SUMMARY OF UNDERLYING CONSTRUCTS

In developing the stochastic optimization model, various techniques were used to reduce the number of scenarios and to model the impact of aircraft reliability on strategic airlift operations.

- Modified repair time - impact of aircraft repair maintenance on MOG. The actual impact is not accurately known and will depend on the experience of Air Force personnel.

- Convolution. This technique is used to reduce the number of scenarios in the model. To determine n, the optimal solution from Throughput II is used as a starting point.

- $\hat{T}_{an}$. The accuracy lost in using $\hat{T}_{an}$ to approximate T_{an} will be examined. As the number of realizations in $\hat{T}_{an}$ increases, the accuracy of the approximation increases, but so does the number of constraints in the stochastic model.

- $MOGPEN_{bt}$. This penalty indirectly reduces the MOG value for an airfield by forcing the airfield to operate at a percentage below MOG, allowing the airfield to have resources available to repair broken aircraft.

- E[Modified Repair Time]. In an airlift operation there will be delays in the delivery of cargo caused by the unexpected grounding of aircraft for repairs. Using the expected modified repair time in the calculation of determining the arrival time of aircraft should improve the model.

There is no method available to check the first construct. On the other hand, the remaining constructs can be verified, not individually, but as a whole by a simulation model. This simulation model will be discussed in Chapter V.

F. MODEL FEATURES

This stochastic model, in addition to the features of Throughput II, accomplishes the following:

- Selection of aircraft routes by anticipating potential bottlenecks in the system.

- Minimizes the number of unreliable aircraft that use capacity limited airfields or airfields that have limited repair capability.

- With the proper choice of $MOGPEN_{bt}$, the optimal solution will achieve a flow of cargo from origin to destination that is not interrupted by the random events of aircraft reliability by forcing an airfield to operate at a percentage of MOG.

IV. BENDERS DECOMPOSITION

As shown in Chapter III, the stochastic optimization model is very large. To solve this optimization model, we use Benders decomposition to convert the large problem into many appropriately coordinated subproblems of manageable size. Section A gives a broad overview of how Benders decomposition works, followed by the theory (Van Slyke and Wets, 1969) in Section B. To solve the stochastic optimization model, an algorithm based on Benders decomposition theory is implemented. An explanation of this algorithm is contained in Section C. Then in Section D, the performance of this algorithm in solving the stochastic optimization model is examined.

A. INTRODUCTION

The stochastic optimization model is inconvenient to solve directly due to the large number of decision variables and structural constraints. Hence, one approach is to adopt a relaxation strategy in which only a few of the constraints are explicitly maintained. Using this approach, the strategy of the decomposition procedure is as follows. The large LP is initially separated into two LPs which are called the Master and the Subproblem. The Subproblem, in turn, separates into one subproblem for each scenario. The Master consists of the general constraints (the deterministic constraints) and the scenario-subproblems consist of special constraints (the stochastic constraints). The decomposition algorithm begins by solving the Master first; the Master then passes down a new set of right-hand-side resources to the Subproblems; the Subproblems are solved and then pass the Master a new set of constraints derived from these resources. The Master is solved again (now consisting of the deterministic constraints and a set of constraints derived from the stochastic subproblems) and the process repeats itself until an optimal solution is obtained. This process is also called a row generation technique.

B. THEORY

To illustrate the ideas underlying Benders decomposition this section considers the special case of a two-stage stochastic linear program with recourse in which there is only one scenario. The extension to the more general case of many scenarios is straightforward. We will also assume the second-stage subproblem is always feasible given any first-stage decisions (called complete recourse). The assumption of complete recourse is realistic whenever second-stage infeasibilities can be modeled via penalty costs. To apply Benders decomposition, complete recourse is not required, but will simplify the discussion. With this special case, the optimization model of interest is

$$\begin{array}{lll} \text{minimize} & cx + fy & \\ \text{subject to} & Ax & = b \\ & -Bx + Wy & = d \\ & x \geq 0, \quad y \geq 0. & \end{array} \tag{4.1}$$

This problem (4.1) is equivalent to

$$\begin{aligned} \text{minimize} \quad & cx + h(x) \\ \text{subject to} \quad & Ax = b \\ & x \geq 0 \end{aligned} \tag{4.2}$$

$$\text{where} \quad h(x) = \begin{aligned} \text{minimize} \quad & fy \\ \text{subject to.} \quad & Wy = d + Bx \\ & y \geq 0. \end{aligned} \tag{4.3}$$

By taking the dual, the recourse function (4.3) can be written as

$$\begin{aligned} h(x) = \text{maximize} \quad & \pi(d + Bx) \\ \text{subject to} \quad & \pi W \leq f\,. \end{aligned} \tag{4.4}$$

Another assumption used in Benders decomposition theory is that the second-stage problem (4.3) is bounded. If it were unbounded for some finite first-stage decision, then by duality theory the corresponding dual must be infeasible. Since the dual feasible region is independent of x, unboundedness can occur in (4.3) only if it occurs for all x. Since the second-stage problem is bounded, the recourse function attains an optimal solution at an extreme point. By enumerating the extreme points of the dual feasible region as $\pi^{(1)},\pi^{(2)},\ldots,\pi^{(L)}$, the recourse function (4.4) is equivalent to

$$h(x) = \underset{1\le i\le L}{\text{maximize}} \quad \pi^{(i)}(d+Bx). \tag{4.5}$$

Upon closer examination of the recourse function (4.5), it can be shown that for each extreme point $\pi^{(i)}$ of the dual feasible region, $\pi^{(i)}d+\pi^{(i)}Bx$, as a function of x, is a hyperplane; $\pi^{(i)}d$ is the intercept and $\pi^{(i)}B$ is the gradient. Each one of the L pieces of $h(x)$ is called a cut and these cuts are supports (or "pieces") of the convex piecewise linear recourse function. In summary, the recourse function is a piecewise linear, convex function generated by the extreme points of the dual feasible region.

As a result of (4.2)/(4.5), the original linear program (4.1) can be written in a form that suggests Benders algorithm:

$$\begin{array}{lll} \text{minimize} & cx+\theta & \\ \text{subject to} & Ax=b & \\ \text{cuts} & -G^{i}x+\theta \ \ge g^{i} & i=1,2,\ldots,L \\ & x\ge 0, & \end{array} \tag{4.6}$$

where $G^{i}=\pi^{(i)}B$ and $g^{i}=\pi^{(i)}d$.

As stated earlier, Benders decomposition is an iterative process. Initially, the Master consists of the deterministic constraints and no cuts while the Subproblem is the recourse function. In the first iteration, the Master problem is solved and passes a new set of resources to the Subproblem. For the given first-stage decision vector, x, the Subproblem will pass up a new cut to the Master problem which represents a "piece" of the piecewise linear function $h(x)$ that is a support at x. Now the Master problem consists of the original deterministic constraints plus this cut. The process repeats itself until optimality is achieved. The algorithm will be computationally efficient when only a small subset of the potentially very large number of cuts needs to be generated.

During any iteration of Benders algorithm, only a subset of the necessary cuts have been added to the Master problem. At each iteration, the Master problem is solved for optimality, denoted $(\hat{x},\hat{\theta})$,where the scalar value $\hat{\theta}$ is an estimate of the recourse function, $h(\hat{x})$. If the set of cuts in the Master problem is complete at $\hat{x}$ then $\hat{\theta} = h(\hat{x})$ and the algorithm terminates with an optimal solution. On the other hand, if the set of cuts in the Master problem is incomplete at $\hat{x}$, then $\hat{\theta} < h(\hat{x})$ and a cut is added to the Master problem. As the set of cuts in the Master problem is augmented each iteration, "more informed" first-stage decisions are made. The algorithm terminates when the Master has sufficient information regarding the recourse function in the neighborhood of an optimal first-stage decision.

During the execution of Benders algorithm, an upper and lower bound on the optimal objective function value can be calculated. At any given iteration, the set of cuts in the Master problem provide a outer linearization of the convex recourse function. Therefore, the lower bound on the objective function value is given by $c\hat{x}+\hat{\theta}$. As each iteration progresses, the Master problem becomes more constrained as cuts are added, therefore the lower bounds generated in successive iterations are increasing. On the other hand, the upper bound on the objective function value is given by $c\hat{x}+f\hat{y}$. This upper bound will not necessarily decrease in successive iterations, therefore the algorithm should save the solution that corresponds to the least upper bound generated so far.

At any given iteration, if the lower and upper bounds of the objective function value are equal, the algorithm terminates. On the other hand, if the upper and lower bounds do not match, then the Subproblem passes a new cut to the Master problem and another iteration is completed.

C. ALGORITHM

It was shown in Chapter III, Section C that the stochastic optimization model developed in this thesis has the following form:

$$
\begin{aligned}
&\text{minimize} && cx+\sum_{b}\sum_{t}E_{\omega}h_{bt}(x,\omega) && (4.7)\\
&\text{subject to} && Ax=b\\
& && x\geq 0,
\end{aligned}
$$

where $h_{bt}(x,\omega) = \underset{R_{bt}^{\omega}}{\text{minimize}} \; MOGPEN_{bt} * R_{bt}^{\omega}$

subject to

$$(1) \sum_{u} \sum_{a} \sum_{r \in R_a} \sum_{\substack{t' \in T_{uar} \\ t' + \lceil DTime_{abr} \rceil = t}} \left[MOG\,\mathrm{Re}q_{ab} * \left(GTime_{abr} + MRTime_{abt}^{\omega} \right) / 24 \right] * X_{uart'}$$

$$+ \sum_{a} \sum_{r} \sum_{\substack{t' \in T_{uar} \\ t' + \lceil DTime_{abr} \rceil = t}} \left[MOG\,\mathrm{Re}q_{ab} * \left(GTime_{abr} + MRTime_{abt}^{\omega} \right) / 24 \right] * Y_{art'}$$

$$- R_{bt}^{\omega} \leq MOGCap_b$$

$$(2) \; R_{bt}^{\omega} \geq 0,$$

where the first-stage decision vector, x, is used to represent all decision variables associated with the deterministic Throughput II model (see Appendix A). These include aircraft scheduling, allocation and inventory decisions as well as accounting variables to track deliveries of troops and equipment.

The stochastic optimization model (4.7) is unique because it has $(b*t)$ convex piecewise linear recourse functions. Expanding upon the theory in Section B, model (4.7) can be written in the following form that suggests Benders algorithm:

$$\min \quad cx + \sum_b \sum_t \theta_{bt} \tag{4.8}$$

$$\text{subject to} \quad Ax = b$$

$$\text{cuts} \quad -G_{bt}^i x + \theta_{bt} \geq g_{bt}^i \qquad \forall\ b,\ t,\ i=1,2,\ldots L_{bt},$$

where G_{bt}^i is the cut gradient and g_{bt}^i is the cut intercept. The calculation for the cut gradient and intercept expressions are contained in Appendix C. Application of Benders algorithm proceeds as follows (see Figure IV-1 for the algorithm):

1. Initialization

A constant called *TOLER* is initialized to a value greater than zero (typically, about 10^{-4}). This constant is used to compare the objective function value upper and lower bound, denoted $\bar{z}$ and $\underline{z}$ respectively. Also, this step initializes θ_{bt} to zero for all airfields b and time periods t; and sets the objective function value upper bound $\bar{z}$ to a very large number.

2. Master Problem

The Master problem is solved and the optimal solution is denoted $(\hat{x},\hat{\theta}_{bt})$. Since $\hat{\theta}_{bt}$ will be less than or equal to the associated recourse cost, the objective function value lower bound can be calculated as

$$\underline{z} \leftarrow c\hat{x} + \sum_{b} \sum_{t} \hat{\theta}_{bt} \,. \tag{4.9}$$

3. Subproblem

The first-stage decision solution $\hat{x}$ is passed to the subproblem. The recourse function is solved, obtaining the value of R_{bt}^{ω} for all airfields b, time periods t and scenario ω. With this information, along with the first-stage decision solution, the objective function value $\hat{z}$ can be calculated as

$$\hat{z} \leftarrow c\hat{x} + \sum_{b} \sum_{t} \sum_{\omega} p_{bt}^{\omega} MOGPEN * R_{bt}^{\omega} \,. \tag{4.10}$$

Since the upper bound $\bar{z}$ does not necessarily decrease in successive iterations, a comparison is made between $\bar{z}$ and $\hat{z}$; if $\hat{z} < \bar{z}$ then the upper bound is set equal to the current objective function value $\hat{z}$ and the associated solution $\hat{x}$ is saved.

4. Optimality Check

A comparison is made between the upper and lower bounds on the optimal objective function value. Specifically, if $\bar{z} - \underline{z} \leq TOLER * \underline{z}$ then the algorithm terminates. If not, the algorithm proceeds to Step 5.

5. Addition of Cuts

The following set of cuts is added to the Master problem

$$-G_{bt}^{i}x + \theta_{bt} \geq g_{bt}^{i} \quad \forall \; b,\, t \tag{4.11}$$

and the algorithm solves the Master problem (go to Step 2). Again, see Appendix C for the details of this calculation.

```
Step 1   Define $TOLER \geq 0$
         Initialize set of cuts with $\theta_{bt} \geq 0 \;\; \forall \; b,t$
         $\bar{z} \leftarrow +\infty$
Step 2   Solve the master problem and obtain $(\hat{x}, \hat{\theta}_{bt})$
         $\underline{z} \leftarrow c\hat{x} + \sum_{b}\sum_{t}\hat{\theta}_{bt}$
Step 3   Calculate $R_{bt}^{\omega} \;\; \forall \; b,t,\omega$
         $\hat{z} \leftarrow c\hat{x} + \sum_{b}\sum_{t}\sum_{\omega} p_{bt}^{\omega}\, MOGPEN * R_{bt}^{\omega}$
         If $\hat{z} < \bar{z}$ then $\bar{z} \leftarrow \hat{z}$ and $x^{*} \leftarrow \hat{x}$
Step 4   If $\bar{z} - \underline{z} \leq TOLER * \underline{z}$ then stop. $x^{*}$ is solution.
Step 5   Augment the set of cuts with $-G_{bt}^{i}x + \theta_{bt} \geq g_{bt}^{i} \;\; \forall \; b,t$
         Goto Step 2.
```

Figure IV-1: Benders Decomposition Algorithm.

D. PERFORMANCE

The algorithm was coded using the FORTRAN 77 programming language and IBM's Optimization Subroutine Library (IBM, 1991).

To improve the efficiency of the algorithm, two techniques are used. Each time the algorithm reaches Step 5 (see Figure IV-1), the algorithm will pass $(b*t)$ cuts to the Master problem, resulting in possible redundant constraints. To remedy this potential problem, a cut for airfield b and time period t is passed to the Master problem only if $\hat{\theta}_{bt} < E_{\omega}h_{bt}(\hat{x},\omega)$.

The second technique is an application of Jensen's inequality. For a fixed first-stage decision $\hat{x}$ the recourse functions $h_{bt}(\hat{x},\omega)$ are convex functions of the modified repair time vector, MRTime_{abt}. Due to Jensen's inequality, the expected recourse function $E_{\omega}h_{bt}(\hat{x},\omega)$ is bounded below by the second-stage cost when we substitute $E[\mathit{MRTime}_{abt}]$ for the random ground times. As stated in Chapter III, Section B.5, the stochastic optimization model uses the original deterministic MOG constraints. By using the expected value of the modified repair time random variable, in addition to the standard ground time, the deterministic MOG constraints provide a "more informed" first-stage decision in the earlier iterations of the algorithm. Then in subsequent iterations, cuts derived from the stochastic MOG constraints "correct" for this optimism.

Using the algorithm shown in Figure IV-1, the stochastic optimization model solves in under 20 minutes on a IBM RS6000 model 590 workstation (the deterministic model, using GAMS, solves in under two minutes), obtaining a near-optimal solution ($TOLER$ = .01). With $MOGPEN_{bt}$ = .04 for all airfields b and time periods t, the algorithm terminates in only nine iterations. As shown in Chapter III, Section C, the stochastic optimization model has a total of 357,591 constraints. By using Benders algorithm, the first iteration started with 7,671 constraints (the deterministic constraints of Throughput II) and the last iteration only had an additional 1,087 stochastic cuts.

V. SIMULATION

This thesis describes two optimization models of the strategic airlift system, a deterministic and stochastic optimization model. Both models produce a recommended deployment schedule, based on four sets of decision variables (X_{uart}, Y_{art}, $Allot_{abt}$ and $Release_{abt}$. See Appendix A for more details.). For example, one decision variable, X_{uart}, will require five C5 aircraft to fly from the onload airfield Travis to the offload airfield Riyadh starting in period two carrying Unit Bravo. Then another decision variable, Y_{art}, will have these five C5 aircraft fly back from Riyadh to Travis starting in period three. However this recommended deployment schedule was developed by an optimization model with its associated modeling techniques and assumptions. So, can this deployment schedule be executed by the Air Mobility Command of the United States Air Force? To address this question, a discrete-event simulation model of the strategic airlift system is developed. The simulation model attempts to execute a given deployment schedule and allows the user to analyze the recommended schedule.

For this thesis the simulation model is used to execute the deployment schedule from the optimization models. The output from the simulation model is then used both to analyze the deterministic and stochastic models and also to compare their proposed deployment schedules. Just as important, examining the results of an optimization model via a simulation model can provide useful feedback with respect to the validity of various linear programming modeling assumptions. The methodology used to develop the simulation model is described in Section A. Section B gives a broad overview of the simulation model followed by a more detailed discussion in Section C. Then in Section D, the performance of this simulation model is examined.

A. METHODOLOGY

The simulation is a discrete-event simulation model of the strategic airlift system. Given a deployment schedule, the simulation will attempt to execute this schedule. Two key concepts are employed in the model. The first key concept is a "blind" execution of the proposed deployment schedule. For example, all aircraft will fly specific routes, even if that route is congested at downstream airfields and a better route is available. This "blind" execution will allow analysis of the proposed schedule. A properly planned deployment schedule will have aircraft land and immediately be serviced. On the other hand, a poorly planned schedule will result in delays as aircraft wait for service at congested airfields.

The second key concept is the ability to execute the simulation model with various options. One option allows the user to turn "ON" or "OFF" the MOG constraint at each airfield. When the MOG constraint is "OFF," all aircraft will land and immediately be serviced even when the MOG limit at that airfield is exceeded. On the other hand, the MOG constraint "ON" option enforces the MOG limit at each airfield. The ability to turn the MOG constraint "ON" or "OFF" will allow the user to examine the effect of a MOG constraint on the proposed schedule. A second option allows the user to run the simulation model with aircraft reliability "ON" or "OFF. When the aircraft reliability option is "OFF," aircraft never break upon landing; on the other hand the aircraft reliability "ON" option allows for the possibility for aircraft to break. This option will allow the user to analyze the effect of aircraft reliability on the proposed deployment schedule.

B. OVERVIEW

This section gives a broad overview of the simulation model. A more detailed discussion is contained in Section C.

1. Deployment Schedule

The simulation model executes a given deployment schedule. Therefore the simulation does not need to determine which route a loaded aircraft will fly, or the amount of cargo to load onto an aircraft. These decisions have been determined by the optimization models and are represented by four types of decision variables. Therefore this deployment schedule given by four types of decision variables is the driving force in the simulation model.

a. Airlift Mission Orders (X Orders)

One type of decision variable is the actual airlift missions flown by the aircraft from onload to offload airfields. This decision vector $[X_{uart}]$ states how many aircraft of type a will transport unit u starting in period t using route r. To convert this variable to the simulation model, there will be X_{uart} individual X orders.[2] Each individual X order will arrive at airfield *b* in time period *t*. The X order requires one aircraft of type *a* to transport cargo and/or troops from unit *u* along route *r*. When the X order arrives at airfield *b* in time period *t*, one of two outcomes will occur. If there is an available aircraft of type *a* at airfield *b*, this X order is assigned to the aircraft. Once this assignment is made, the aircraft will execute the order. On the other hand if there are no aircraft of type *a* available at airfield *b*, the X order is assigned to a first-in, first-out (FIFO) queue at airfield *b*. When an aircraft of type *a* becomes available at airfield *b* at a later time, the X order is assigned to the aircraft.

Once the X order is assigned, the aircraft will consume MOG resources and onload cargo for unit *u*. Once the onload is complete the aircraft will take off and fly along route *r*. If the required route has any enroute airfields the aircraft will land at the enroute airfield and the repair time random variable is used to determine the required repair time if the reliability option is "ON." The aircraft will then consume MOG resources to refuel the aircraft and, if needed, repair the aircraft. When complete the aircraft will take off and continue to fly along route *r*. Eventually the aircraft will land at the offload airfield. Once again the repair time random variable is used to determine the required repair time if the reliability option is "ON." The aircraft will consume MOG resources to offload the cargo and, if needed, complete repairs. When complete one of two outcomes will occur. If there is an old Y order (to be discussed in the next subsection) at airfield *b* for an aircraft of type *a*, the order will be assigned to the aircraft and the aircraft will execute the order. On the other hand if no Y order exists, the aircraft will remain in inventory at airfield *b* and wait for a future order to arrive.

---

[2] The decision variables are continuous numbers. The simulation model will convert the continuous number to an integer by (1) rounding, (2) rounding down or (3) rounding up the solution from the optimization model. The simulation model allows the user to choose one of these three options for each type of decision variable X_{uart}, Y_{art}, $Allot_{abt}$ and $Release_{abt}$.

b. Aircraft Recovery Orders (Y Order)

Another type of decision variable is the recovery flights of aircraft from the offload airfields back to the onload airfields. This decision vector $[Y_{art}]$ states how many aircraft of type *a* are to fly recovery route *r* starting in period *t*. To convert this variable to the simulation model, Y_{art} individual Y orders will arrive at airfield *b* in time period *t*. Each individual Y order requires an aircraft of type *a* to fly recovery route *r*. When the Y order arrives at airfield *b* in time period *t*, one of two outcomes will occur. If there is an aircraft of type *a* at airfield *b*, this Y order is assigned to the aircraft. Once this assignment is made, the aircraft will execute the order. On the other hand, if there are no aircraft of type *a* available at airfield *b*, the Y order is assigned to a FIFO queue at airfield *b*. When an aircraft of type *a* becomes available at airfield *b* at a later time, the Y order is assigned to the aircraft.

Once the Y order is assigned, the aircraft will immediately take off and fly along route *r*. Eventually the aircraft will land at the onload airfield and the repair time random variable is used to determine the required repair time if the reliability option is "ON." At this point, one of two outcomes will occur depending on whether the aircraft is mission capable or not. If the aircraft is mission capable, the aircraft will either be assigned an X order that arrived at the airfield at an earlier time for an aircraft of type *a*, or the aircraft will be placed in inventory at airfield *b* and wait for an X order to arrive. On the other hand, if the aircraft requires repair work, the aircraft will consume MOG resources to complete repairs. Once the repairs are complete, the aircraft will either execute an X order that arrived at the airfield at an earlier time for an aircraft of type *a*, or the aircraft will be placed in inventory at airfield *b* and wait for a future X order to arrive.

c. *Aircraft Allotment Orders (A Order)*

The third type of decision variable is the allotment of aircraft to the airlift system. This decision vector [$Allot_{abt}$] states how many aircraft of type *a* to add to onload airfield *b* in time period *t*. During the execution of the simulation, $Allot_{abt}$ individual A orders will be executed in time period *t*. Each A order will add one aircraft of type *a* to onload airfield *b* in time period *t*. When the aircraft arrives in time period *t* at onload airfield *b*, one of two outcomes will occur. If there is an old X order for an aircraft of type *a* at airfield *b*, the order is assigned to the aircraft and the order is executed. On the other hand, if there are no X orders at airfield *b*, the aircraft will be placed in inventory at airfield *b* until an X order arrives.

d. Aircraft Release Orders (R Order)

The last group of decision variables releases aircraft from the airlift system. This decision vector [$Release_{abt}$] states how many aircraft of type *a* are to be released from onload airfield *b* in time period *t*. During the execution of the simulation, $Release_{abt}$ individual R orders will arrive at onload airfield *b* in time period *t*. One of two outcomes will occur. If there exists an available aircraft of type *a* at airfield *b*, the aircraft will leave the airlift system. On the other hand, if there are no aircraft of type *a* available, the order will wait in a queue (FIFO) until an aircraft of type *a* arrives. When an aircraft of type *a* is available, the assignment of the R order takes precedence over an X order.

2. Aircraft Service Priority

One very important modeling issue is the priority for processing aircraft at the airfields. When the MOG constraint option is "OFF," aircraft will land and immediately consume MOG resources as required. But when the MOG constraint option is "ON," some rule must be used to prioritize the aircraft. When aircraft require service (onload, refuel, offload, or aircraft repairs) in the simulation, the aircraft are assigned either a code-one or code-two service priority. The distinction is as follows. When an airfield is MOG constrained, code-one aircraft are always serviced first, followed by code-two aircraft. Also, if a code-one aircraft arrives at a MOG constrained airfield, the simulation will interrupt any code-two aircraft that are consuming MOG resources. For example, suppose an airfield is MOG constrained with a specified number of code-two aircraft consuming MOG resources. If a code-one aircraft lands, the required number of code-two aircraft will be interrupted to allow the code-one aircraft to consume MOG resources.

The assignment of service priorities is as follows. A code-two service priority is always assigned to an aircraft that is non-mission capable (i.e., breaks) upon landing. This will allow code-one aircraft to immediately be serviced and complete their mission. A code-one service priority can be assigned to an aircraft in one of two ways. First, if an aircraft is mission capable upon landing and requires MOG resources, the aircraft is assigned a code-one service priority. The second way an aircraft can be assigned a code-one service priority is as follows. If an aircraft is non-mission capable after landing, a code-two service priority is assigned to the aircraft. Now if the airfield is MOG constrained, the aircraft will wait until all code-one aircraft have been serviced, but this can be an indefinite amount of time since aircraft may continue to arrive. Therefore, one assumption in the simulation model is that all aircraft must be repaired within the repair time random variable realization. With this assumption, the simulation will upgrade a code-two aircraft to code-one after a predetermined amount of repair/waiting time elapses.

To determine the time a code-two aircraft is upgraded to code-one is as follows. When an aircraft lands at an airfield and breaks, the required repair time realization and associated modified repair time realization are known. The repair time realization is the number of hours until the aircraft is mission capable. The modified repair time realization is the hours of MOG resources required to repair this aircraft. As stated earlier, the simulation will attempt to repair all aircraft within the repair time random variable realization. Thus if the repair time realization is two hours and the modified repair time realization is two hours, this aircraft must consume MOG resources immediately to complete repairs. In this case the aircraft, upon breaking, is assigned a service priority of code-one, not code-two. On the other hand if the repair time random variable realization is 72 hours and the modified repair time realization is 30 hours, this aircraft can wait at most 42 hours prior to consuming MOG resources. In this case, the aircraft will transition to code-one after waiting 42 hours to consume MOG resources.

In general the difference between the remaining repair time and the remaining modified repair time is called the Slack Time. When the aircraft initially breaks, the Slack Time is equal to the repair time realization minus the modified repair time realization. When the aircraft is waiting to consume MOG resources, the remaining repair time is decreasing linearly with time. As the remaining repair time decreases linearly with time, eventually the Slack Time will equal zero. At this time the aircraft will be upgraded from code-two to code-one. On the other hand if the aircraft is consuming MOG resources, the remaining repair time and modified repair time are both decreasing linearly with time. Therefore while the aircraft is consuming MOG resources, it is impossible for the aircraft to be upgraded from code-two to code-one.

Two criteria are used to determine when a non-mission capable aircraft becomes mission capable. First, the modified repair time requirement must be satisfied. This can only be accomplished when the aircraft consumes MOG resources for the required amount of time. The second criteria is the repair time must be satisfied. This is equivalent to requiring that the Slack Time equal zero. The second criteria is needed for the following reason. If a code-two aircraft immediately consumes MOG resources upon landing, the modified repair time criteria has been met. But the modified repair time is a measure of the impact of this broken aircraft on MOG. In reality, the plane will not be available until the repair time requirement has been satisfied.

3. Output

The purpose of the simulation model is to examine the feasibility of an airlift deployment schedule and to compare the solution provided by the deterministic and stochastic solutions. With this in mind, two key outputs from the simulation model are the delivery profile and landing profile graphs. The delivery profile graph plots the total amount of cargo and passengers delivered (in stons) to the offload bases versus time. The landing profile graph plots the total number of aircraft landings at the offload bases versus time. These graphs are extremely useful in examining the proposed schedule from an optimization model and to compare the solution from the deterministic and stochastic optimization models. This will be discussed further in Chapter VI.

Another useful output from the simulation model is a graph of the number of aircraft that are waiting for or consuming MOG resources versus time for each airfield.

A quick look at the graph allows the user to determine at what time an airfield is MOG constrained and by how much.

C. MODEL

This section gives a detailed description of the simulation model.

1. Input Data

The required input data to run the simulation model is as follows:

- Airfield data. To execute a deployment schedule, the only data required is the latitude/longitude and MOG value of each airfield. The latitude/longitude is used to compute the distance between each airfield.

- Aircraft data. The only data required is the block speed and the deterministic ground time for the each aircraft type. The block speed will allow the simulation model to correctly compute the time for an aircraft to fly between airfields.

- Aircraft reliability. This input file gives the repair time random variable realizations, modified repair times realizations and associated probabilities for C5, C17 and C141 aircraft. All other aircraft do not break.

- Routes. This data is used in conjunction with the optimization solution so the simulation can properly route aircraft from airfield to airfield.

- Optimization solution. The driving force for the simulation is the deployment schedule developed by the optimization models. This consist of four groups of decision variables X_{uart}, Y_{art}, $Allot_{abt}$, $Release_{abt}$.

2. Queuing System for MOG Consumption

There are a total of eight queues used to properly process aircraft. They are described below and a diagram is shown in Figure V-1.

a. Taxi Queue

If the airfield is MOG constrained, aircraft are placed in the Taxi (FIFO) Queue. When a unit or partial unit of MOG becomes available, the next aircraft in the Taxi Queue will enter the Code-1F, Code-1P, Code-2F or Code-2P Queue. If the next aircraft in line has a service priority of code-two, the aircraft immediately enters the Hold-In Queue and time now counts toward the repair time requirement.

b. Code-1F Queue

This queue is for aircraft with a service priority of code-one. Each aircraft is consuming a full unit of MOG and time is counting toward both the modified repair time and repair time requirement (if any). The aircraft will remain in this queue for an amount of time equal to the modified repair time plus the standard ground time. When services are complete, the aircraft will then exit.

c. Code-1P Queue

This queue is for aircraft with a service priority of code-one, but the aircraft is only consuming a partial unit of MOG. At any given time, there will be at most one aircraft in this queue. Time is counting toward both the modified repair time and repair time requirement (if any). If an aircraft leaves the Code-1F Queue at a MOG constrained airfield, the aircraft in the Code-1P Queue, if it exists, will enter the Code-1F Queue and consume a full unit of MOG. It is also possible for an aircraft to remain in the

Code-1P Queue long enough to satisfy the modified repair time requirement plus the standard ground time, at which time it will exit.

d. Code-2F Queue

This queue is for aircraft with a service priority of code-two where each aircraft is consuming a full unit of MOG. Time is counting toward both the modified repair time and repair time requirement. The aircraft will remain in this queue for an amount of time equal to the modified repair time plus the standard ground time. When services are complete the aircraft will enter the Hold-Out Queue until the repair time requirement is satisfied. If at any time a code-one aircraft needs to consume resources and the airfield is MOG constrained, the required number of aircraft in the Code-2F Queue, if any, will be interrupted and enter the Hold-In Queue.

e. Code-2P Queue

This queue is for an aircraft with a service priority of code-two but this aircraft is only consuming a partial unit of MOG. At any given time, there will be at most one aircraft in this queue. Time is counting toward both the modified repair time and repair time requirement. When an aircraft leaves the Code-2F or Code-1F Queue and the airfield is MOG constrained, the aircraft in the Code-2P Queue, if it exists, will enter the Code-2F Queue and consume a full unit of MOG. It is also possible for the aircraft to satisfy the modified repair time requirement plus the standard ground time while the aircraft is in this queue, at which time the aircraft will enter the Hold-Out Queue until the repair time requirement is satisfied. If at any time a code-one aircraft needs to consume resources and the airfield is MOG constrained, the aircraft in the Code-2P, if there is one, will be interrupted and enter the Hold-In Queue.

f. Hold-In Queue

Only code-two aircraft enter this queue. Time is counting toward the repair time requirement. When a unit or partial unit of MOG becomes available in either the Code-2F or Code-2P Queue, an aircraft, if any, will leave the Hold-In Queue and enter the appropriate queue. If the Slack Time reaches zero for any aircraft while in this queue, the aircraft will be upgraded to code-one and enter the Upgrade Queue.

g. Upgrade Queue

Only aircraft that have been upgraded from code-two to code-one enter this queue. From the Upgrade Queue aircraft will enter either the Code-1F or Code-1P Queue when a unit or partial unit of MOG becomes available.

h. Hold-Out Queue

This queue is for code-two aircraft that have satisfied the modified repair time requirement, but have yet to satisfy the repair time requirement. The aircraft is not consuming MOG resources. When the repair time requirement has been satisfied, the aircraft will exit.

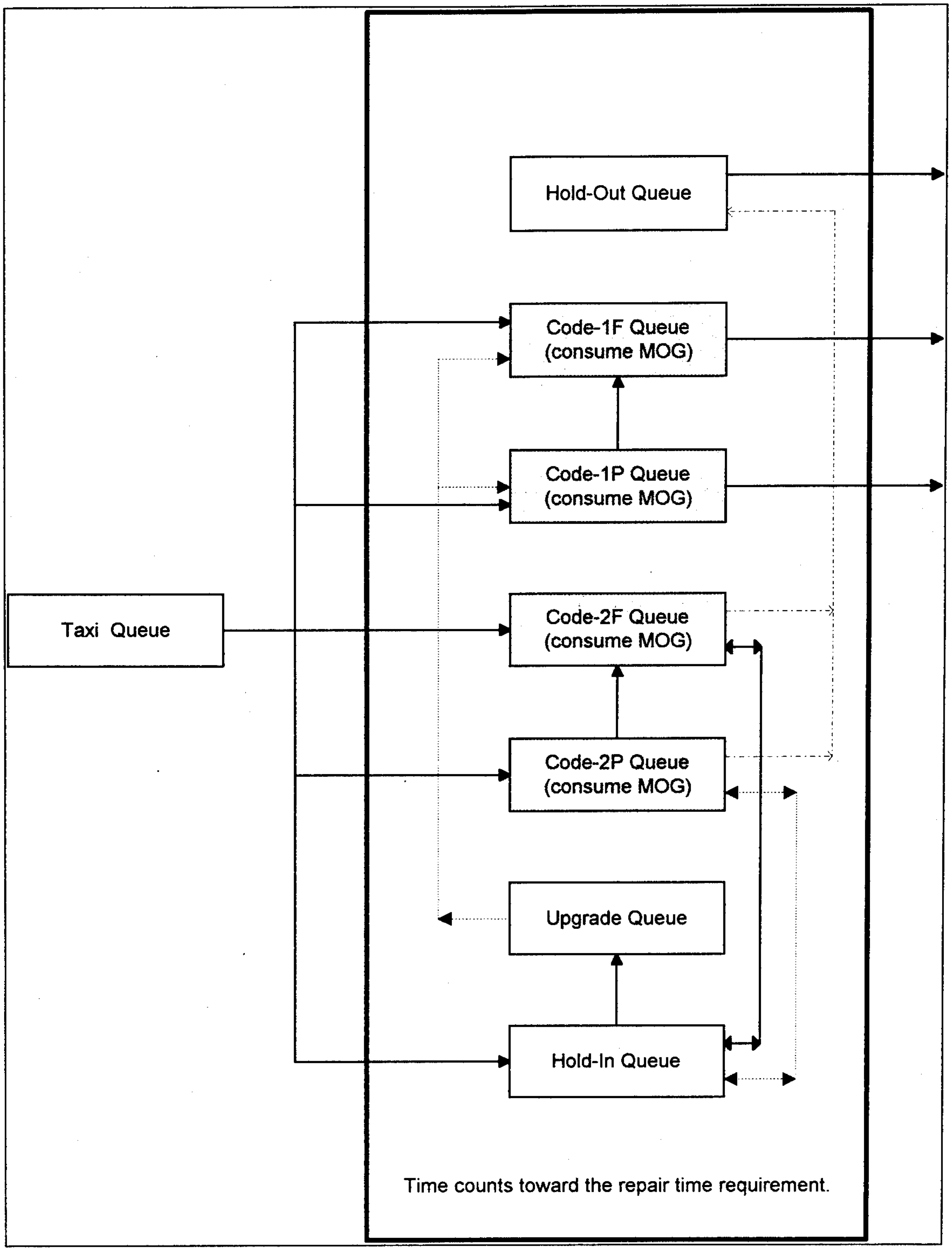

Hold-Out Queue
Code-1F Queue (consume MOG)
Code-1P Queue (consume MOG)
Taxi Queue
Code-2F Queue (consume MOG)
Code-2P Queue (consume MOG)
Upgrade Queue
Hold-In Queue
Time counts toward the repair time requirement.

Figure V-1: Queuing System for MOG Consumption.

i. Priorities Among Queues

When an aircraft has completed consuming MOG resources, the simulation will use the following priority to determine the next aircraft to consume MOG resources:

1. Code-1P Queue
2. Code-2P Queue
3. Upgrade Queue
4. Taxi Queue
5. Hold-In Queue.

3. State Variables

In simulation models, state variables are a collection of variables necessary to describe a system at a particular time. For this simulation model the state variables can be grouped into two general categories.

a. Aircraft State Variables

Aircraft Type	Defines the aircraft type; e.g., C5, C17, C130, C141, 747C, 747P, DC10.
ID Number	The identification number of the aircraft. This allows the simulation to track each individual aircraft.
Cargo	This variable is true if the aircraft has cargo onboard and fully loaded, and false otherwise.
Cargo ID	The name of the unit being transported by the aircraft.
Break Code	Defines the required repair time for the aircraft (MC, two, six, ten, fourteen, twenty, thirty-six, sixty, and seventy-two).
Status	Defines the stage of the aircraft in the system. They are as follow: airborne, onload, refuel, offload, repair, inventory, pre-airlift (the aircraft has not yet entered the airlift system) and

post-airlift (the aircraft has been released from the airlift system).

Service Priority	When an aircraft needs to consume MOG resources, this state variable gives the location of the aircraft in the queuing system for MOG consumption (see Figure V-1). The possible values are taxi, code-1F, code-1P, code-2F, code-2P, hold-in, hold-out, upgrade and NA (the aircraft does not require MOG services).
Current Base	The name of the airfield where the aircraft is currently located.
Entered Service	The time the aircraft left the Taxi Queue and entered the Code-1F, Code-1P, Code-2F, Code-2P or the Hold-In Queue.
MOG Time	The total hours of MOG resources required by this aircraft. This includes the standard ground time and the modified repair time.
Cum Time	The current amount of MOG resources consumed by the aircraft.

b. Airfield State Variables

Name	The name of the airfield.
Lat	The latitude position of the airfield.
Long	The longitude position of the airfield.
MOGReq	The narrow-body equivalent requirement for each aircraft type for this airfield.
MOG	The narrow-body MOG limit.
Current	The number of narrow-body equivalent aircraft consuming MOG resources.
Req'd	The number of narrow-body equivalent aircraft that are consuming MOG resources and those aircraft that need to consume MOG resources. Specifically, this is the narrow body equivalent sum of aircraft whose service priority state variable is equal to code-1F, code-1P, code-2F, code-2P, taxi, and upgrade. This is used as a measure of how much the MOG value is being exceeded.

R Orders	There is a FIFO queue for each aircraft type. Each queue holds old R orders that still need to be executed.
X Order	There is a FIFO queue for each aircraft type. Each queue holds old X orders that still need to be executed.
Y Order	There is a FIFO queue for each aircraft type. Each queue holds old Y orders that still need to be executed.
Inv	Lists all aircraft that are being held as inventory at this airfield.
Taxi	Lists the aircraft assigned to the Taxi Queue for that airfield.
Code1F	Lists the aircraft assigned to the Code-1F Queue for that airfield.
Code1P	Lists the aircraft assigned to the Code-1P Queue for that airfield.
Code2F	Lists the aircraft assigned to the Code-2F Queue for that airfield.
Code2P	Lists the aircraft assigned to the Code-2P Queue for that airfield.
Upgrade	Lists the aircraft assigned to the Upgrade Queue for that airfield.
HoldIn	Lists the aircraft assigned to the Hold-In Queue for that airfield.
HoldOut	Lists the aircraft assigned to the Hold-Out Queue for that airfield.

4. Events

In simulation modeling events are defined as an instantaneous occurrence that may change the state of the system. The events used in this simulation model are briefly described below and the event diagrams are contained in Appendix E.

Allot Order	This event adds an aircraft to an onload airfield. The aircraft will either remain in inventory at the airfield or be assigned an old X order. If an old X order exists the aircraft will execute that X order. The event diagram is contained in Figure D-1.

Release Order	This event attempts to remove an aircraft from the airlift system. If the aircraft is not available, the order will enter a FIFO queue at the airfield until the appropriate aircraft arrives. The event diagram is contained in Figure D-2.
X Order	This event attempts to execute a new X order. If there is an available aircraft, the X order is assigned to the aircraft and the aircraft will execute the X order. If there are no aircraft available, the X order is entered in a FIFO queue at the specified airfield. The event diagram is contained in Figure D-3.
Y Order	This event attempts to execute a new Y order. If there is an available aircraft, the Y order is assigned to the aircraft and the aircraft immediately takes off. On the other hand, if there are no available aircraft, the Y order is entered in a FIFO queue at the specified airfield. The event diagram is contained in Figure D-4.
Land	This event lands an aircraft at an airfield. If the reliability option is "ON," the repair time random variable is used to determine the amount of time required to repair the aircraft. If the aircraft just arrived at an onload airfield and is mission capable, the aircraft is available to execute an X order. If the aircraft needs to refuel, offload or complete repairs, the aircraft will consume MOG resources. The event diagram is contained in Figure D-5.
Ready-1F	A specific aircraft in a Code-1F Queue just completed consuming MOG resources and will exit. The aircraft will continue to execute its assigned order. The event diagram is contained in Figure D-7.
Ready-1P	A specific aircraft in a Code-1P Queue just completed consuming MOG resources and will exit. The aircraft will continue to execute its assigned order. The event diagram is contained in Figure D-7.
Ready-2F	A specific aircraft in a Code-2F Queue just completed consuming MOG resources. The aircraft will enter the Hold-Out Queue. The event diagram is contained in Figure D-8.
Ready-2P	A specific aircraft in a Code-2P Queue just completed consuming MOG resources. The aircraft will enter the Hold-Out Queue. The event diagram is contained in Figure D-8.

Ready-Hold	A specific aircraft in the Hold-Out Queue has satisfied the repair time requirement and will continue its mission. The event diagram is contained is Figure D-10.
Transition	A specific aircraft in the Hold-In Queue is upgraded from a service priority of code-two to code-one. The aircraft is removed from the Hold-In Queue and entered in the Upgrade Queue. The event diagram is contained in Figure D-11.

5. Execution Options

There are numerous options available in the simulation. The main options are described below.

a. Time to Execute Orders

As stated earlier, the optimization models use discrete time periods, but the simulation model has a continuous time spectrum. For example, an optimization model solution might add four C5 aircraft to airfield *b* in day two. But in the simulation, day two extends from 24 to 48 hours. At what time should the simulation model add these aircraft? To address this issue the simulation model provides the user with two options. Using the first option, the simulation model will have all new orders arrive at the beginning of the corresponding time period. Using the second option, the simulation model will uniformly spread the new orders throughout the time period. For example, suppose the optimization model requires that four C5 aircraft be added to airfield *b* in day two. Using the second option, the simulation will add the four C5 aircraft at hours 24, 30, 36 and 42.

b. Converting the Continuous Decision Variable Solutions to Integers

The four groups of decision variables are used as input data for the simulation model. But the variables are continuous numbers and the simulation must use integer numbers. Therefore for each one of the four decision variables, the user has three options. They are (1) round-down, (2) round, and (3) round-up. This will be discussed further in Chapter VI.

c. MOG constraint

If this option is "ON" the simulation model will enforce the MOG constraint at each airfield. On the other hand if this option is "OFF," all aircraft will consume MOG resources as needed even when the MOG limit at that airfield is exceeded.

d. Aircraft Reliability

When the aircraft reliability option is "OFF," aircraft never break upon landing; on the other hand the aircraft reliability "ON" option allows for the possibility for aircraft to break.

D. PERFORMANCE

The simulation model is coded using Borland Pascal with Objects (Borland, 1992). If the simulation model is run with the reliability option "OFF," only one run is required and this run takes about 12 seconds (486 DX2 66 Hz computer). If the simulation model is run with the reliability option “ON,” multiple runs are required until steady state is achieved and this takes about six hours.

VI. ANALYSIS

This chapter reports the results of using the simulation model as a tool to analyze the deterministic and stochastic optimization models. For all analyses a base scenario developed by the U.S. Air Force Studies and Analyses Agency is used. Section A discusses two separate issues that must be resolved before the simulation can be used as an analytical tool. Then in Section B the simulation model is used to examine the effect of the deterministic MOG efficiency factor. Section C reports the results of using the simulation model to analyze the deterministic model. Finally, Section D compares the deterministic and stochastic optimization models using the simulation.

A. COUPLING OF THE OPTIMIZATION MODELS WITH SIMULATION

Section A explains default settings for two options that may be set by the user.

1. Converting the Continuous Decision Variable Solution to Integers

As stated earlier, the four types of decision variables passed to the simulation from the optimization model are continuous numbers. To convert the continuous decision variable solution to integers, the simulation model will round-down, round, or round-up depending on the option chosen by the user. The following criteria is used to choose the correct setting for this option for each type of decision variable. Since the simulation model is used to study the deployment schedule recommended by the optimization models, the simulation model needs to execute the same number of airlift missions (X orders) and recovery flights (Y orders). For example if the total number of airlift missions in the deterministic model is 100.40, the simulation model needs to execute a total of 100 or 101 X orders. Once this is achieved, there will still be an error associated with the individual rounding of the decision variable solution. For example, one airlift mission might require 12.5 C5 aircraft to airlift unit *u* using route *r* starting in time period *t*, but if the round-up option is used, this will be translated to 13 C5 aircraft.

To examine the impact of this issue, the deterministic optimization model is solved with a MOG efficiency factor equal to one. Using this solution as input, the simulation model is executed using the three different round options for each one of the four types of decision variables. When the round-down option is used for the X orders, a total of 3,624 X orders are generated in the simulation model compared to the original number of 3,734.40 from the optimization model. In this case the simulation will attempt to execute 3,624 X orders and is 110.40 short of the required amount. Using the round-down option for the Y orders, the simulation attempts to execute 3,280 Y orders compared to 3,388.70 from the optimization model. In this case the Y orders are short by 108.7. In summary the round-down option results in a total of 219.1 X and Y orders that are not considered for execution in the simulation model. The same analysis can be performed for the round and round-up option. See Table VI-1 for a summary of the results.

	Round-Down	Round	Round-Up
Sum of X Orders - Simulation	3624	3734	3836
Sum of X Orders - Optimization	3734.4	3734.4	3734.4
X Order Excess	-110.4	-0.4	101.6
Sum of Y Orders - Simulation	3280	3387	3499
Sum of Y Orders - Optimization	3388.7	3388.7	3388.7
Y Order Excess	-108.7	-1.7	110.3
X and Y Order Excess	-219.1	-2.1	211.9
Number of X Orders Not Executed	12	3	5
Number of Y Orders Not Executed	10	2	14
Number of X and Y Orders Not Executed	22	5	19
Corrected X and Y Order Excess	-241.1	-7.1	192.9

Table VI-1: Simulation Performance versus Round Option. The deterministic optimization model (*MOGEff* = 1.00) is used as input into the simulation model.

There is a consequence of converting the continuous decision variable solution to integers. The optimization models use aircraft balance constraints at onload and offload airfields to ensure there are aircraft available to fly the airlift missions and recovery flights. When the decision variable solution is converted from a continuous number to an integer, this aircraft balance might no longer exist. In the simulation, for example, an X order will not be executed if there was no corresponding Y order to return the required aircraft type to that airfield. Therefore at the completion of one run, the simulation will count the number of X and Y orders that are not executed. The results are also summarized in Table VI-1.

As shown in Table VI-1, when the round-down option is used for X and Y orders the simulation model needs to execute an additional 241.10 X and Y orders to properly model the flow of aircraft through the airlift infrastructure. The round-up option executes an extra 192.90 X and Y orders. However, by using the round option, the simulation is only short by 7.10 orders. Therefore when using a MOG efficiency factor equal to one, the round option closely matches the optimization model. Similar results are obtained if different MOG efficiency factors are used. Therefore all analyses use the round option for X and Y orders.

In converting the continuous decision variable solution associated with the allotment and release of aircraft, all analyses use the round option. In most cases there is no difference between the number of aircraft used in the optimization and the simulation models.

2. Arrival of Orders

A second simulation option that the user may control regards scheduling the arrival of orders within the time periods given by the optimization models. As stated earlier the optimization models use discrete time periods, but the simulation model has a continuous time spectrum. To address this issue, the simulation model gives the user two options. The first option will have all orders arrive at the very beginning of the time period, denoted "0000." The second option will have the orders arrive uniformly throughout the time period, denoted "uniform." Table VI-2 illustrates the option used in all analyses for each decision variable.

Decision Variable	Option
X_{uart}	uniform
Y_{art}	0000
$Allot_{abt}$	uniform
$Release_{abt}$	uniform

Table VI-2: Arrival of Orders Option.

The reason for this choice is as follows. The simulation should use a realistic schedule within the time periods. In airlift operations the Air Force attempts to maintain sufficient spread between departure times so as to minimize congestion at onload, enroute and offload airfields. This same type of concept is applied in the simulation model by using the "uniform" option for A (If there are old X orders at the airfield, aircraft will be assigned the X orders and request MOG services to onload the cargo.) and X orders. On the other hand, when the aircraft arrive at the destination and offload their cargo in airlift operations, they will immediately take off and return to an onload airfield. Therefore the "0000" option is used for the arrival of Y orders at offload airfields. Finally, the "uniform" option is used for the arrival of R orders to uniformly spread the departure of aircraft from the airlift system throughput the time period.

The options shown in Table VI-2 result in the simulation behaving in a consistent fashion. Using the simulation model with the MOG constraint option "OFF" and the reliability option "OFF," all aircraft will immediately consume MOG as required and experience no delays caused by breakage. This results in a verbatim execution of the airlift schedule. By examining simulation output graphs similar to Figure VI-1 for all airfields, analysts can determine when the given schedule is exceeding the capacity of an airfield. By varying the arrival options, the user can determine the best combination for all decision variables. For example the "0000" option for all four decision variables yields the solid line in Figure VI-1. In this case the capacity of Travis airfield is exceeded on days six, eight and eleven. Upon closer examination of the scenario, aircraft are allotted to Travis airfield on day six and eleven at midnight. But there are old X orders existing at the airfield. Therefore the X orders are immediately assigned to the aircraft and the capacity of the airfield is exceeded as the aircraft onload their cargo. Thus the solid line in Figure VI-1 will result in delays as aircraft wait for service when the MOG constraint is enforced. On the other hand, the dash line is obtained by using the recommended options and results in a more uniform arrival rate and consumption of MOG.

B. DETERMINISTIC MOG EFFICIENCY FACTOR

As stated in Chapter II, aircraft turnaround times for onloading and offloading cargo and enroute refueling are assumed to be known constants in the Throughput II model, although they are naturally stochastic. This ignores the fact that deviations from the given service time can cause congestion on the ground. To offset the optimism of this assumption, Throughput II uses an efficiency factor (<1) in the formulation of MOG consumption constraints to lessen the impact of randomness. This section estimates a reasonable value for this MOG efficiency factor.

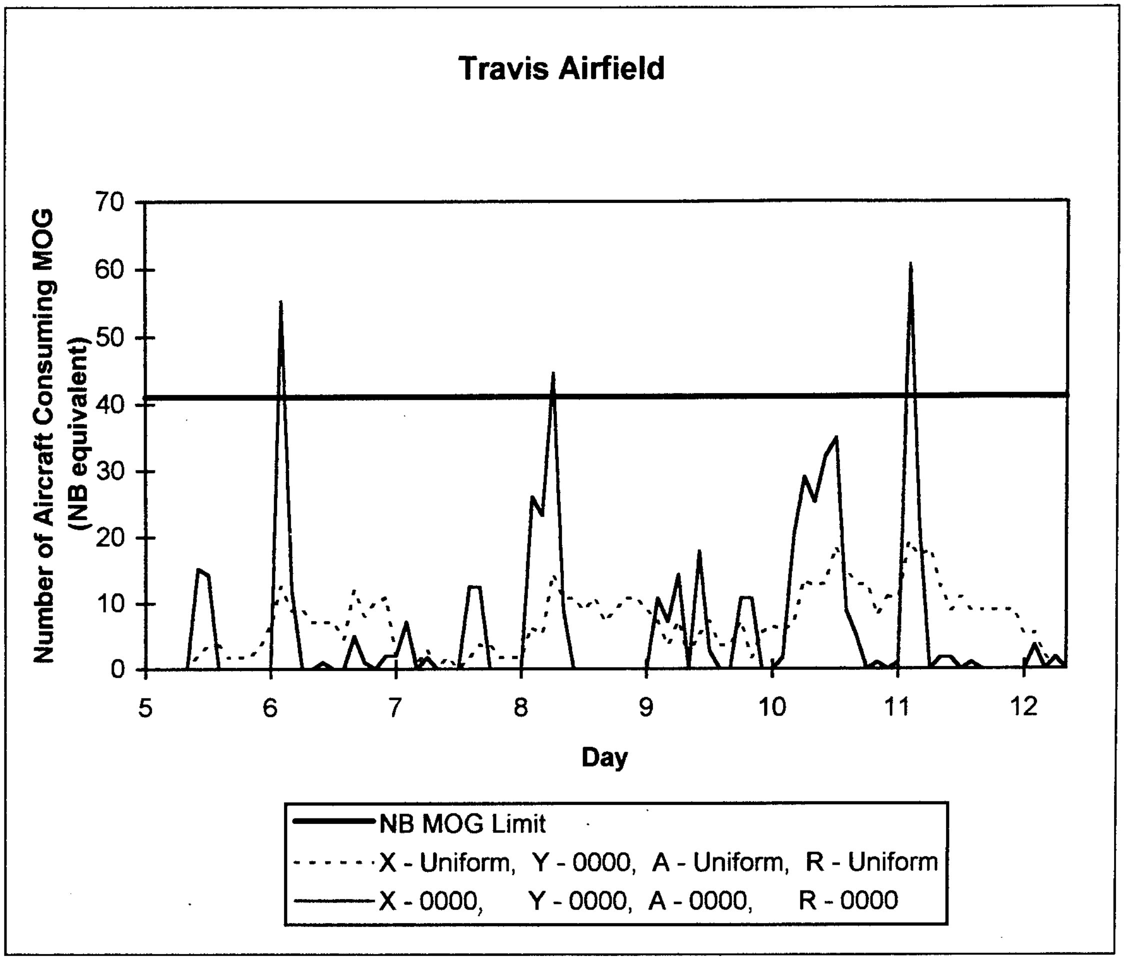


Figure VI-1: Simulation Output Graph of Travis Airfield. The deterministic model (*MOGEff* = 1.00) is used as input into the simulation model. The simulation is then run with the following options: MOG "OFF," reliability "OFF."

1. Methodology

The following approach is used to estimate a reasonable value for the deterministic MOG efficiency factor.

a. Optimization Model

Solve the deterministic optimization model with a specified MOG efficiency factor and obtain the delivery profile graph (Figure VI-2).

b. Effect of Discrete Time Periods

Using the deterministic solution as input, run the simulation model with the following options: reliability "OFF" and MOG "OFF." The delivery profile graph obtained from this simulation run will differ from step (a); the difference between the two delivery profile graphs shows the effect of the discrete time periods of the optimization model. Examining Figure VI-2, the simulation completed the delivery of cargo on day 31 compared to day 26 for the optimization model.

c. Effect of Aircraft Reliability

Perform another simulation run with the following options: reliability "ON" and MOG "OFF." This delivery profile graph takes into account aircraft reliability and reflects the minimum delay (since the MOG constraint is "OFF") to deliver all the cargo as aircraft spend more time on the ground to complete repairs (see Figure VI-2).

d. Effect of MOG Constraint

Perform a simulation run with the following options: reliability "ON" and MOG "ON." The delivery profile graph now takes into account the uneven arrival of aircraft and the limited resources available to repair the non-mission capable aircraft. This effect can be adjusted by changing the MOG efficiency factor. As shown is Figure

VI-2, the end result is that all cargo will not be delivered until about day 43. One interesting observation is that on day 26, the optimization model has delivered all the cargo, but the simulation still has 35,000 stons to deliver, or approximately 17 percent.

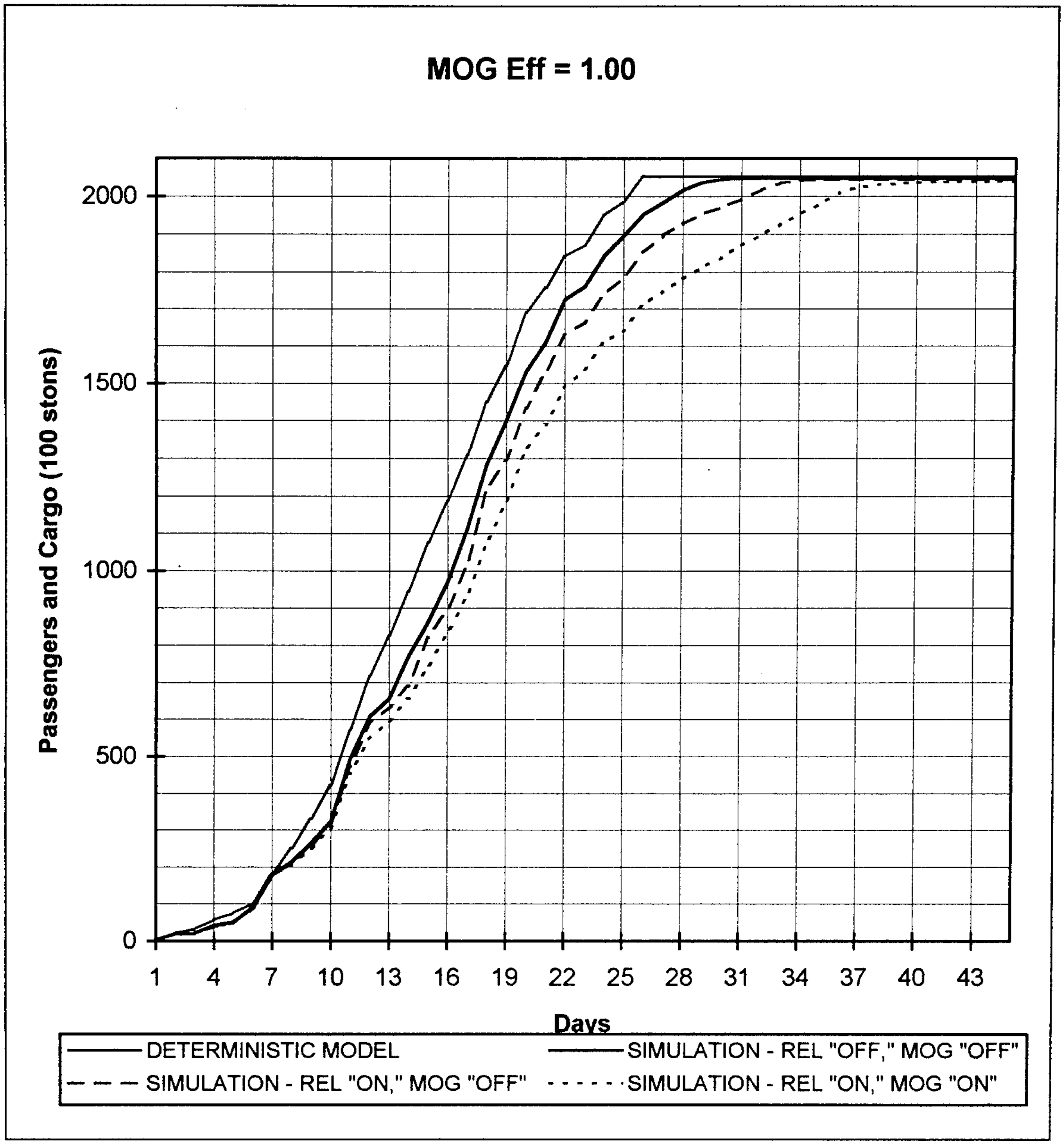


Figure VI-2: Simulation Delivery Profile Comparison. The deterministic model (MOGEff = 1.00) is used as input into the simulation model.

2. Results

Using this approach, Appendix F contains six similar graphs with MOG efficiency factors of 0.50, 0.60, 0.70, .080, 0.90 and 1.00. The effect of the MOG constraint is negligible when a MOG efficiency factor of 0.50 is used. But operating the airfields at half capacity is unrealistic because the airlift system would be dramatically underutilized. On the other hand, the effect of the MOG constraint is significant when a MOG efficiency factor of 1.00 is used. By examining the graphs in Appendix F, one can conclude that the best MOG efficiency factor is approximately 0.70 or 0.80. Also if the six delivery profile graphs (MOG "ON" and Reliability "ON") are compared, the graphs corresponding to MOG efficiency factors of 0.70 and 0.80 deliver more cargo when the airlift infrastructure is MOG constrained.

Another output from the simulation model is the landing profile at offload airfields versus time. This graph plots the total number of landings at the offload airfields versus time. Using the same approach as the delivery profile graphs, the landing profile graphs in Appendix D are obtained. Once again, the best MOG efficiency factor to use is either 0.70 or 0.80.

C. ANALYSIS OF DETERMINISTIC OPTIMIZATION MODEL

The assumptions and limitations of the deterministic optimization model were discussed in Chapter II, Section B. One limitation of the model is the discrete time periods. Specifically, the model enforces the MOG constraint for each airfield using the discrete time periods. With one-day time periods, the model can route aircraft in a manner that causes local congestion. For example, all aircraft could arrive at an airfield within a small time window instead of being dispersed over an entire day. In reality this would cause local congestion, even though the model's representation of aircraft handling capacity is observed. Another limitation is that the model rounds the time (to the nearest day) at which an aircraft arrives at origin and destination airfields.

To examine this issue of discrete time periods, the simulation model can be used. One of the simulation output graphs is the number of aircraft that are consuming MOG resources and those aircraft that need to consume MOG (specifically those aircraft in the Taxi and Upgrade Queue) resources versus time for each airfield. Two steps are used. First perform a simulation run with the reliability option "OFF" and the MOG constraint option "OFF" and obtain the output graph. Examining Figure VI-3 the following points are illustrated:

- Uneven aircraft arrival. The graph indicates a very non-uniform aircraft arrival rate at Mildenhall airfield.

- MOG constraint. During the time period from day 10 to 25, the airfield is MOG constrained according to the deterministic optimization model solution report. But from day 11 to 12 the simulation model exceeds the one day MOG limit, but the time period from day 12 to 13 does not. This reflects the error caused by rounding in the optimization model.

Next perform a simulation run with the reliability option "OFF" and the MOG constraint option "ON." Now the simulation model will continuously enforce the MOG constraint, unlike the one-day average enforcement by the optimization model. Examining Figure VI-3 the following points are illustrated:

- Up to day seven, both simulation runs track. But then they diverge even though the airfield is not MOG constrained. This reflects the fact that other airfields are MOG constrained, thus delaying the arrival of aircraft to Mildenhall airfield.

- From day 23 to 26 the airfield is exceeding the MOG capability of the airfield. At one point about 80 NB equivalent aircraft are on the ground waiting and consuming MOG resources.

Another simulation output that is used to examine the deterministic optimization model is the delivery profile graph. The following steps are taken (see Figure VI-4):

1. Obtain the delivery profile graph from the deterministic optimization model.

2. Using the deterministic model solution as input into the simulation, perform a simulation run with the following options: reliability "OFF" and MOG "OFF." This difference in the delivery profile graphs illustrates the effect of the discrete time periods. In this case, the graph indicates that, on average, there is a difference of about 10,000 stons.

3. Perform a simulation run with the following options: reliability "OFF" and MOG "ON." The difference in the delivery profile graphs illustrate the effect of limited MOG resources at the various airfields. In this case, the difference is negligible.

4. Perform a simulation run with the following options: reliability "ON" and MOG "ON." The difference in the delivery profile graph illustrates two effects: the delay caused by aircraft actually breaking and spending more time on the ground and the delay, if any, as the aircraft wait for the extra MOG resources required to fix the aircraft. This additional delivery profile graph can not be used to justify the proper choice of the MOG efficiency factor since the delay caused by aircraft simply breaking is not related to the MOG efficiency factor.

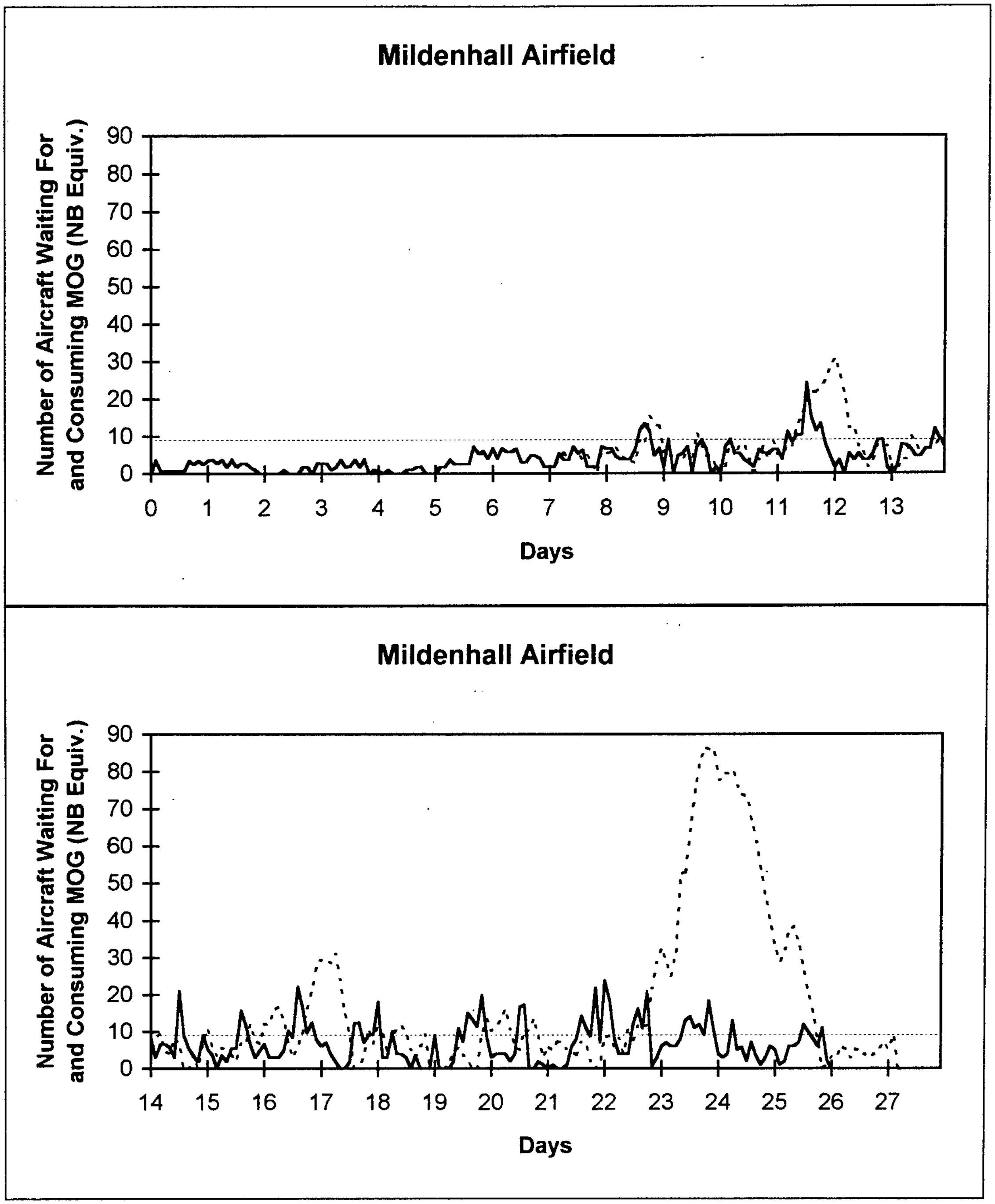


Figure VI-3: Simulation Output Graph of Mildenhall Airfield. The deterministic model (*MOGEff* = 0.80) is used as input into the simulation model. The solid line is the simulation run with REL "OFF" and MOG "OFF." The dash line is the simulation run with REL "OFF" and MOG "ON."

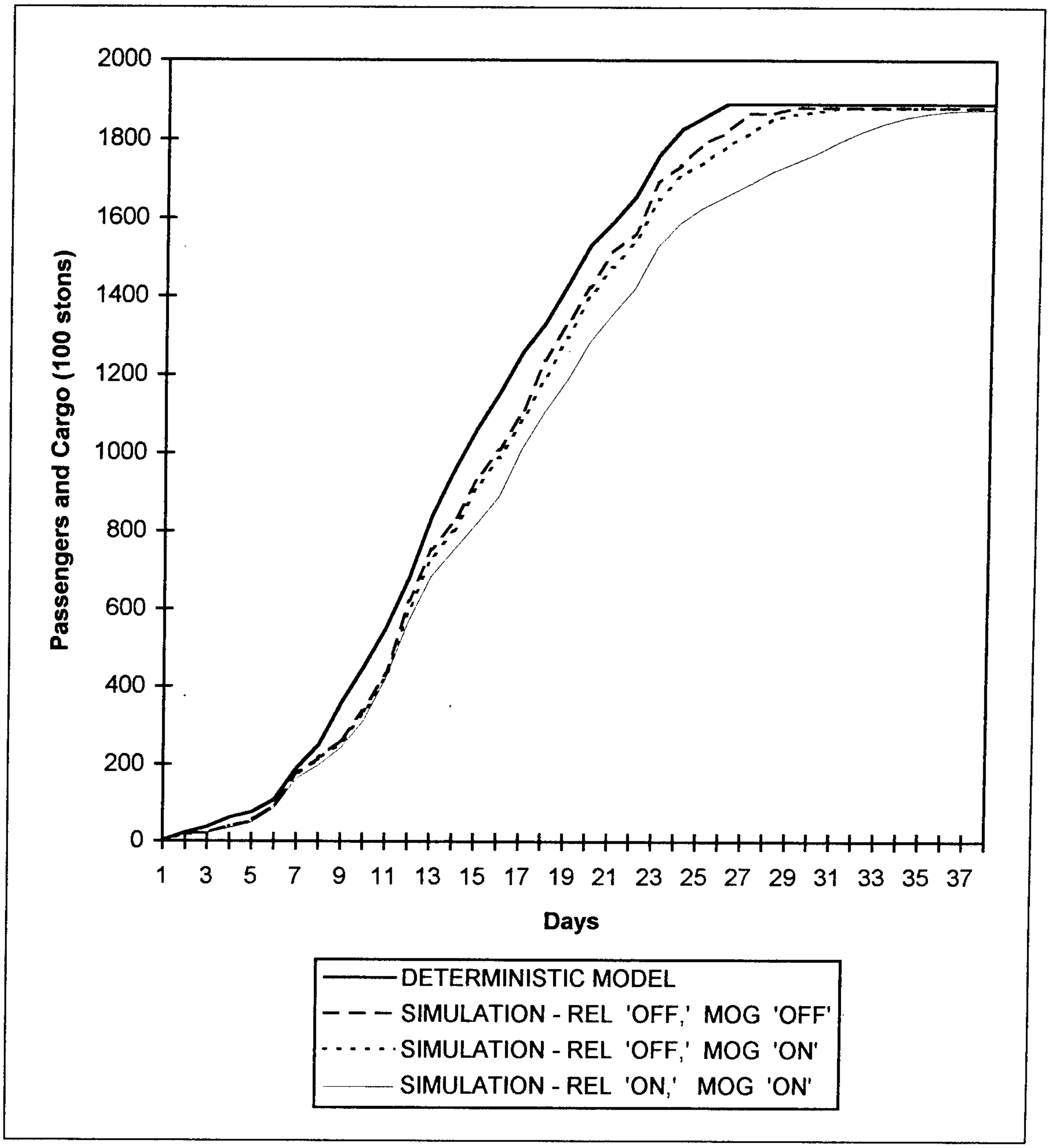


Figure VI-4: Simulation Deliver Profile Comparison. The deterministic model (*MOGEff* = 0.80) is used as input into the simulation model.

D. DETERMINISTIC AND STOCHASTIC OPTIMIZATION COMPARISON

Before comparing the two optimization models, the impact of the MOG penalty in the stochastic optimization model is examined. Using the formulation from Chapter III, Section D, the MOG penalty can range from 0.001 to 0.26. The total amount of cargo delivered as a function of the MOG penalty is illustrated in Figure VI-5.

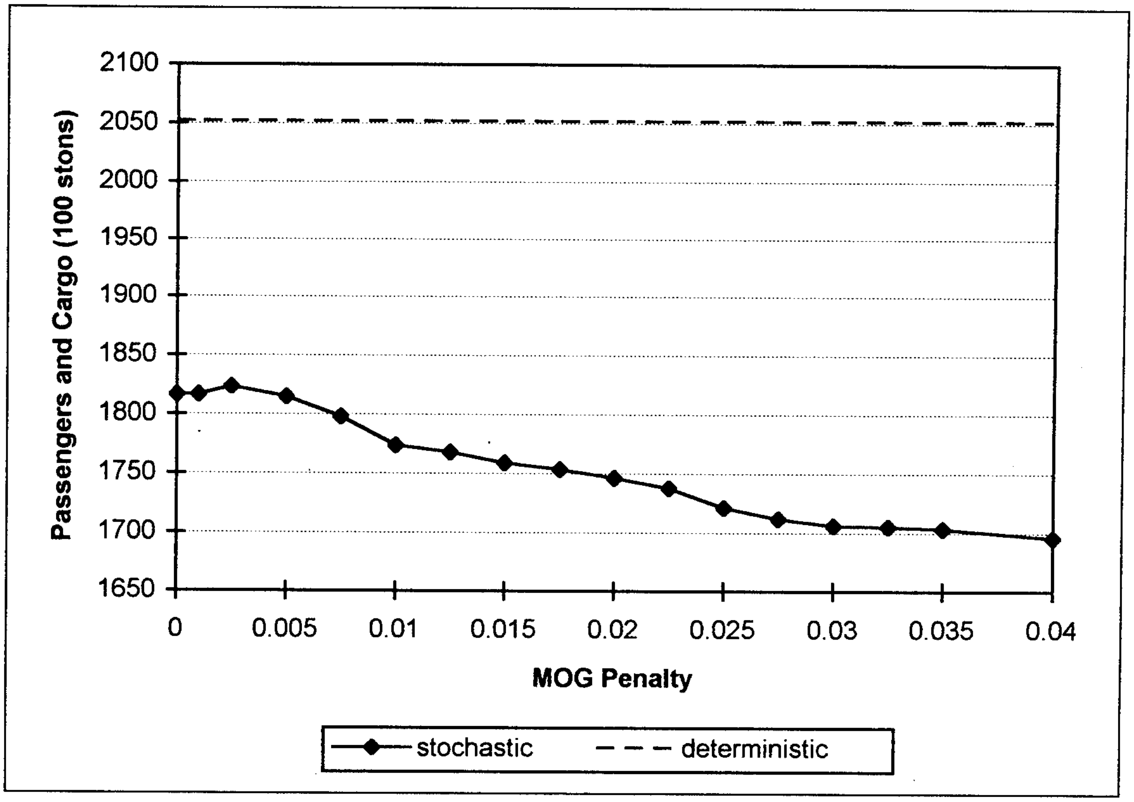


Figure VI-5: Total Cargo and Passengers Delivered by the Stochastic Optimization Model. The deterministic model is solved with a MOG efficiency factor equal to one.

To compare the stochastic and deterministic optimization models the following approach is used. Specify a value for the stochastic MOG penalty and solve the stochastic optimization model. The solution from the stochastic optimization model, specifically the deployment schedule, is used as input into the simulation model. The simulation model is then run with the following options: MOG "ON" and reliability "ON." When the simulation has reached steady state the delivery profile graph is obtained. Using this approach the three delivery profile graphs are obtained using MOG penalties of 0.00, 0.02 and 0.04 for the stochastic optimization model (see Figure VI-6). For comparison the same approach is used for obtaining the delivery profile graph for the deterministic optimization model.

The stochastic optimization model with a MOG penalty equal to zero is equivalent to solving a modified version of the deterministic model. The modified deterministic model uses the expected value of the modified repair time, in addition to the deterministic ground time, for the calculation of MOG consumption. Since the modified repair time is being used in the calculation of the MOG consumption, the MOG efficiency factor needs to be set to one.

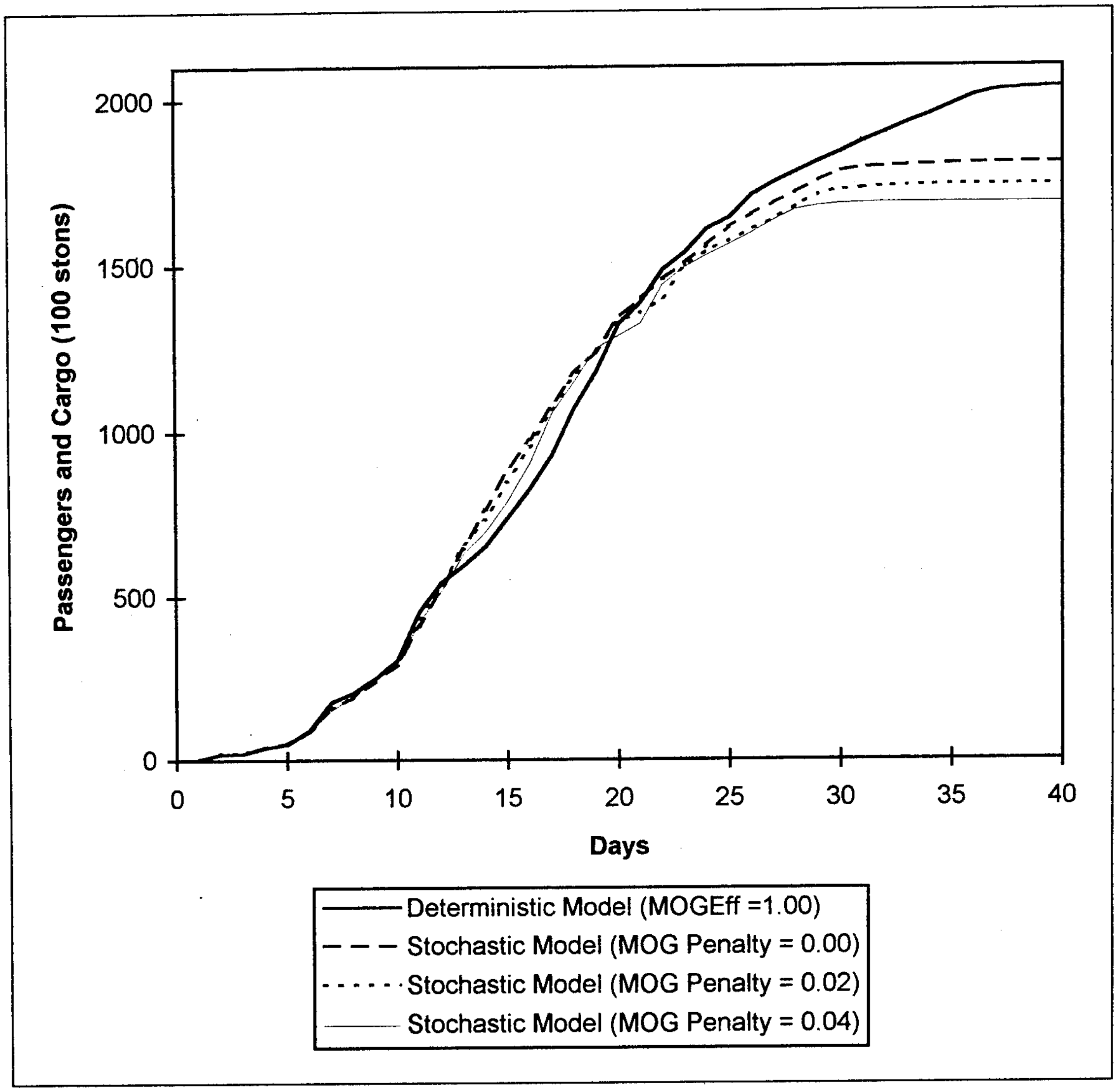


Figure VI-6: Simulation Delivery Profile Comparison. The four delivery profile graphs are obtained by first solving the corresponding optimization model. Then the solution from the optimization model is used as input into the simulation model. The simulation is then run with the following options: MOG "ON" and reliability "ON."

All four optimization/simulation model runs can be characterized as having three distinct phases as observed in Figure VI-6. In the first 12 days of the base scenario, the system is airframe constrained and all four model runs are very similar. But from day 12 to about day 20, the airlift infrastructure is the limiting factor, specifically airfield capacity. During this time period all three stochastic models exceed the deterministic model in the delivery of cargo and passengers. From day 21 and on, the system is in a sustainment phase with diminished demands. Neither airframes nor airfield capacities are critical resources. One interesting point illustrated by the graphs is that the schedules generated by all three stochastic models delivered all the cargo in the simulation model by day 30; but the deterministic model took an 10 extra days, or an additional one-third of the required 30 day time span.

Closer examination of the time period from day 12 to day 20 lends some insight into the power of the stochastic optimization model. When the deterministic optimization model is solved, the model is operating critical airfields at maximum capacity during this time period (day 12 to 20). This will allow the optimization model to deliver the largest amount of cargo. Then the solution from the deterministic optimization model is used as input into the simulation and the simulation model is run with the MOG constraint option "ON" and the reliability option "ON." But when the simulation gets to this time period all it takes is for one aircraft to break at a MOG constrained airfield to have a dramatic impact. This aircraft will need to consume a larger amount of MOG resources resulting in follow-on aircraft waiting on the ground while this aircraft is being repaired. As more and more aircraft break, the problem just multiplies as more and more aircraft wait on the ground for MOG resources.

On the other hand the stochastic optimization model takes a different approach as illustrated by the following three points. First, when the stochastic optimization model is solved, the model will not operate these critical airfields at maximum capacity because the associated recourse costs outweigh the benefits. Therefore the stochastic model will operate the critical airfields at some percentage below their maximum capacity. This will allow extra MOG resources to be available to repair aircraft as they break. Second, since the stochastic optimization model will be delivering less cargo during this time period it will be more selective in determining which units to move. Finally the stochastic optimization model is re-routing aircraft to minimize the number of unreliable aircraft that are using capacity limited airfields. Therefore when the stochastic optimization model solution is used as input into the simulation, the simulation does not experience major bottle-necks at these critical airfields and will exceed the deterministic model in the amount of cargo delivered.

As seen in Figure VI-6 the benefits of the stochastic optimization model take effect when the infrastructure is MOG constrained. In this case, this is from day 12 to about day 20 (see Figure VI-7). During this time period, all three stochastic model runs deliver more cargo than the deterministic model (no matter what number is used for the MOG efficiency factor, the deterministic model deliver less cargo than the stochastic model). Figure VI-7 indicates that as the MOG penalty gets larger the stochastic model delivers less cargo during this time period. But consider the following points:

- As the MOG penalty is increased from 0.00, the delivery profile initially drops off, but by day 17 all three graphs are nearly the same. At this point the diminishing demand begins to take effect and all three graphs approach their final delivery amount. If the airlift infrastructure was MOG constrained over a longer time period, it is possible that a difference might be noted.

- Using the delivery profile graph as a measure of effectiveness does not directly reflect the value of the stochastic optimization model. Another possible measure would be the probability that an airfield exceeds its capacity in a given time period.

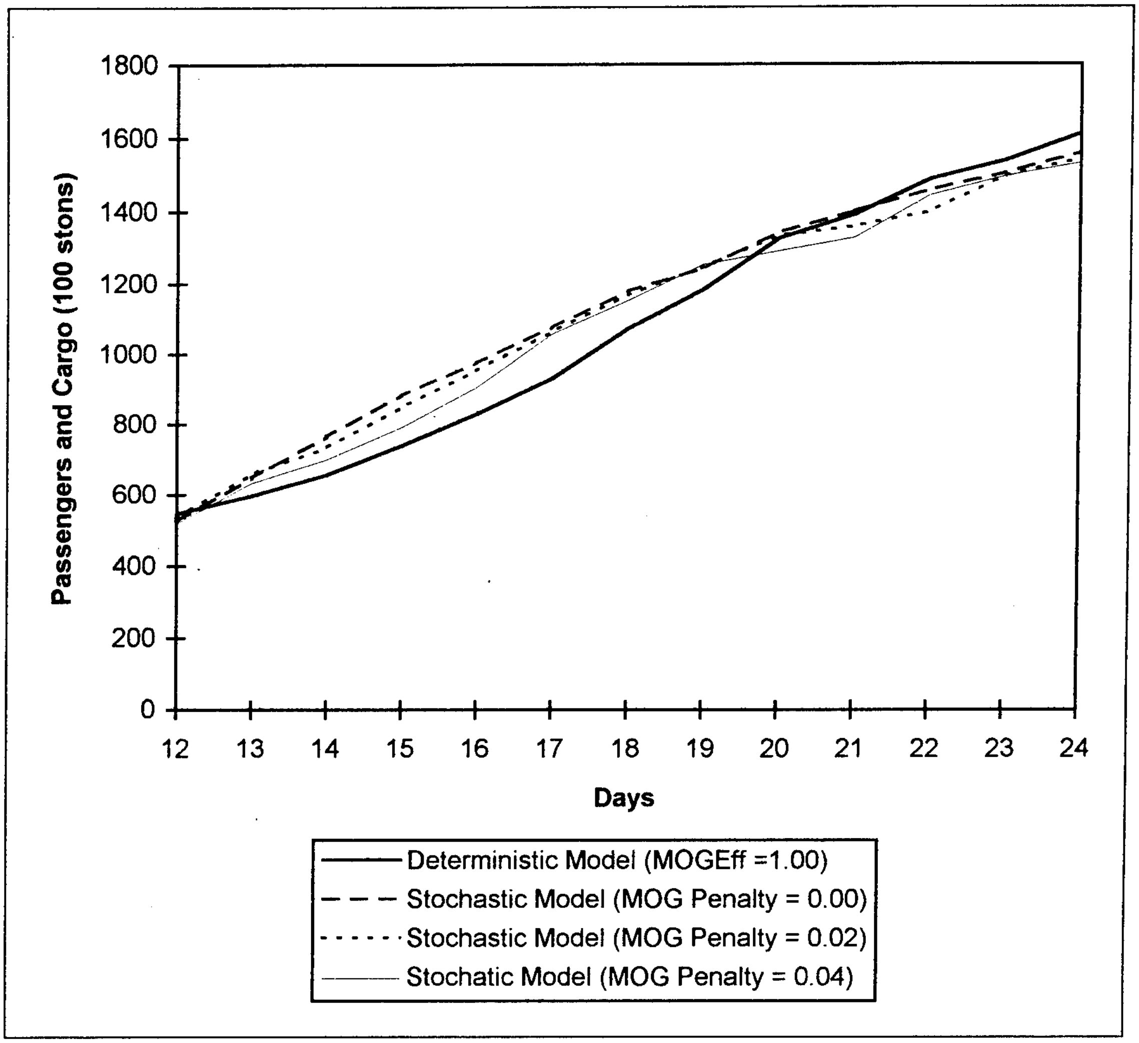


Figure VI-7: Simulation Delivery Profile Graph. The four delivery profile graphs are obtained by first solving the corresponding optimization model. Then the solution from the optimization model is used as input into the simulation model. The simulation is then run with the following options: MOG "ON" and reliability "ON."

Another difference is found between the stochastic and deterministic optimization solutions. Examining the time period from day 12 to day 20, Table VI-3 shows the percent of X orders (airlift missions) flown by each aircraft type.

Model	C5	C17	C141	C130	P747	C747
deterministic	24.1	7.5	37.6	22.6	3.7	4.4
stochastic (pen = 0.00)	25.9	7.6	18.9	34.8	6.3	6.5
stochastic (pen = 0.04)	26.3	8.7	10.7	38.4	8.5	7.4

Table VI-3: Percent of X orders (airlift missions) flown by each aircraft type from day 12 to day 20.

The following points explain the results obtained in Table VI-3.

- Break rate data is not available for the C130, P747, and C747 and therefore were modeled as perfectly reliable aircraft. Relative to this, we have considerable insight as to why the models behave the way they do.

- The stochastic model (MOG penalty = 0.00) differs from the deterministic model only in that the ground times of the C5, C17, and C141 are increased by the expected value of the modified repair time random variable. Therefore the C130, P747 and C747 are, relatively speaking, more attractive in the stochastic model (MOG penalty = 0.00); this is why their usage increases. From Jensen's inequality the expected value will provide an optimistic prediction of an aircraft's performance relative to using the probability distribution. Thus the C130, P747, and C747 are again, relatively speaking, more attractive when we move from the stochastic model (MOG penalty = 0) to the stochastic model (MOG penalty = 0.04); this is why the usage of the perfectly reliable aircraft again increases.

- Why does the C130 appear to displace the C141 as opposed to the C17 or C5 (the other two aircraft with stochastic ground times)? The C130 and C141 can both carry bulk and over but not out-sized cargo. To move the outsized cargo we need the C5 and C17.

- The C130 is a very slow (block speed) aircraft and is not usually even regarded as a strategic airlifter. The results do not suggest that the C130 is the answer to the strategic airlift problem. There are a couple of unique characteristics of this base scenario. First, there is the perfectly reliable assumption for the C130 aircraft.

Second, a large fraction of the total movement in this 30 day scenario is from Ramstein to Saudi Arabia; the fact that this is (in strategic airlift terms) such a short distance makes the C130 more of a player than it might otherwise be. Finally, the fact that the theater is operating at MOG capacity means that the slow block speed does not slow down throughput as much as it might otherwise.

In summary the stochastic optimization model, in addition to the features of Throughput II, accomplishes the following:

- Selection of aircraft routes by anticipating potential bottlenecks in the system.
- Minimizes the number of unreliable aircraft that use capacity limited airfields or airfields that have limited repair capability.
- Operating critical airfields at a percentage below their maximum capacity in order to have excess MOG resources available to repair aircraft as they break. This allows the stochastic optimization model to achieve a flow of cargo to the theater that is not interrupted by the random events of aircraft reliability.

VII. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

A stochastic extension has successfully been added to the Throughput II model to account for aircraft reliability. This stochastic extension results in a two-stage stochastic optimization model with recourse.

The stochastic optimization model can be used to gain broad insights into the strategic airlift system. Some of the broad insights that can be gained by including aircraft reliability are the change in fleet mix and the re-routing of aircraft compared to the deterministic optimization model. But most important of all is the fast turn-around of the stochastic optimization model. This allows quick answers without tying down a massive amount of manpower or computer resources.

In addition the airlift infrastructure has also been modeled by a discrete-event simulation model. This allows the user to analyze the recommended deployment schedule from the deterministic and stochastic optimization models. The simulation will also allow the user to analyze a given deployment schedule and examine the effects of MOG and aircraft reliability.

B. RECOMMENDATIONS

The following are some recommendations based on the analysis completed in Chapter VI.

1. MOG Efficiency Factor

Using the simulation model to analyze the deterministic model, Chapter VI estimated a reasonable value of 0.80 for the MOG efficiency factor.

2. Modification to Deterministic Optimization Model

The current form of the deterministic optimization model is overly optimistic. Even though the MOG efficiency factor can be used to lessen the impact of random ground times, it does not take into account aircraft reliability. To account for aircraft reliability, the deterministic model can be modified by using the expected value of the modified repair time, in addition to the standard ground time, for the calculation of MOG consumption. Since the modified repair time is being used in the calculation of the MOG consumption, the MOG efficiency factor needs to be set to one. Now the model takes into account aircraft reliability and the analysts can examine the effect of aircraft reliability on the mix of aircraft used and the re-routing of aircraft.

3. Stochastic Optimization Model

The next step is to solve the stochastic optimization model with a larger data set. This thesis examined a 30 day scenario, but during this time span the airlift infrastructure was MOG constrained only for a period of eight days. The effects of the stochastic optimization should be more evident if a larger problem is used for analysis.

APPENDIX A. THROUGHPUT II MODEL

The following is a brief summary of Throughput II (Morton *et. al.*, 1995).

A. INDICES

u	indexes units, e.g., 82nd Airborne
a	indexes aircraft types, e.g., C5, C141
t,t'	index time periods
b	indexes all airfields (origins, enroutes and destinations)
i	indexes origin airfields
k	indexes destination airfields
r	indexes routes

B. INDEX SETS

1. Airfield Index Sets

B	set of available airfields
$I\subseteq B$	origin airfields
$K\subseteq B$	destination airfields

2. Aircraft Index Sets

A	set of available aircraft types
$A_{bulk}\subseteq A$	aircraft capable of hauling bulk size cargo
$A_{over}\subseteq A_{bulk}$	aircraft capable of hauling over-sized cargo
$A_{out}\subseteq A_{over}$	aircraft capable of hauling out-sized cargo

Bulk cargo is palletized on 88 x 108 inch platforms, which can fit on any plane. Over-sized cargo is non-palletized rolling stock; it is larger than bulk cargo and can fit on a C141, C5 or C17. Out-sized cargo is very large non-palletized cargo that can fit into a C5 or C17 but not a C141.

3. Route Index Sets

R	set of available routes
$R_a \subseteq R$	permissible routes for aircraft a
$R_{ab} \subseteq R_a$	permissible routes for aircraft a that use airfield b
$R_{aik} \subseteq R_a$	permissible routes for aircraft a that have origin i and destination k
$DR_i \subseteq R$	delivery routes that originate from origin i
$RR_k \subseteq R$	recovery routes that originate from destination k

4. Time Index Set

T	set of time periods
$T_{uar} \subseteq T$	possible launch times of sorties for unit u using aircraft a and route r

The set T_{uar} covers the allowed time window for unit u, which starts on the unit's available-to-load date and ends on the unit's required delivery date, plus some extra time up to the maximum allowed lateness for the unit.

C. GIVEN DATA

1. Movement Requirements Data

$MovePAX_{uik}$	Troop movement requirement for unit u from origin i to destination k
$MoveUE_{uik}$	Equipment movement requirement in short tons (stons) for unit u from origin i to destination k
$ProBulk_u$	Proportion of unit u cargo that is bulk-sized
$ProOver_u$	Proportion of unit u cargo that is over-sized

$ProOut_u$	Proportion of unit u cargo that is out-sized

2. Penalty Data

$LatePenUE_u$	Lateness penalty (per ston per day) for unit u equipment
$LatePenPAX_u$	Lateness penalty (per soldier per day) for unit u troops
$NoGoPenUE_u$	Non-delivery penalty (per ston) for unit u equipment
$NoGoPenPAX_u$	Non-delivery penalty (per soldier) for unit u troops
$MaxLate$	Maximum allowed lateness (in days) for delivery
$Preserve_{at}$	Penalty (small artificial cost) for keeping aircraft a in mobility system at time t

3. Cargo Data

$UESqFt_u$	Average cargo floor space (in sq. ft.) per ston of unit u equipment
$PAXWt_u$	Average weight of a unit u soldier inclusive of personal equipment

4. Aircraft Data

$Supply_{at}$	Number of aircraft of type a that become available at time t
$MaxPAX_a$	Maximum troop carriage capacity of aircraft a
$PAXSqFt_{ua}$	Average cargo space (in sq. ft.) consumed by a unit u soldier fro aircraft a
$ACSqFt_a$	Cargo floor space (in sq. ft.) of aircraft a
$LoadEff_a$	Cargo space loading efficiency (<1) for aircraft a. This accounts for the fact that it is not possible in practice to fully utilize the cargo space.

$URate_a$	Established utilization rate (flying hours per aircraft per day) for aircraft a

5. Airfield Data

$MOGCap_b$	Aircraft capacity (in narrow-body equivalents) at airfield b
$MOGReq_{ab}$	Conversion factor to narrow-body equivalents for one aircraft of type a at airfield b
$MOGEff$	MOG efficiency factor (<1), to account for the fact that it is impossible to fully utilize available MOG capacity due to randomness of ground times

6. Aircraft Route Performance Data

$MaxLoad_{ar}$	Maximum payload (in stons) for aircraft a flying route r
$GTime_{abr}$	Aircraft ground time (due to onload or offload of cargo, refueling, maintenance, etc.) needed for aircraft a at airfield b on route r
$DTime_{abr}$	Cumulative time (flight time plus ground time) taken by aircraft a to reach airfield b along route r
$FltTime_{ar}$	Total flying hours consumed by aircraft a on route r
$CTime_{ar}$	Cumulative time (flight time plus ground time) taken by aircraft a on route r
$DaysLate_{uart}$	Number of days late unit u's requirement would be if delivered by aircraft a via route r with mission start time t

D. DECISION VARIABLES

1. Sortie Variables

X_{uart}	Number of aircraft a that airlift unit u via route r with mission start time during period t

Y_{art}	Number of aircraft a that recover from a destination airfield via route r with start time during period t

2. Aircraft Allocation and De-allocation Variables

$Allot_{ait}$	Number of aircraft a that become available at time t that are allocated to origin i
$Release_{ait}$	Number of aircraft a available at origin i in time t that are not scheduled for any flights from then on

3. Aircraft Inventory Variables

H_{ait}	Number of aircraft a inventoried at origin i at time t
HP_{akt}	Number of aircraft a inventoried at destination k at time t
$NPlanes_{at}$	Number of aircraft a in the air mobility system at time t

4. Airlift Quantity Variables

$TonsUE_{uart}$	Total stons of unit u equipment airlifted by aircraft a via route r with mission start time during period t
$TPAX_{uart}$	Total number of unit u troops airlifted by aircraft a via route r with mission start time during period t

5. Elastic (Nondelivery) Variables

$UENoGo_{uik}$	Total stons of unit u equipment with origin i and destination k that is not delivered in the prescribed time frame
$PAXNoGo_{uik}$	Number of unit u troops with origin i and destination k who are not delivered in the prescribed time frame

E. OBJECTIVE

minimize

$$\sum_{u}\sum_{a}\sum_{r\in R_a}\sum_{t\in T_{uar}} LatePenUE_u * DaysLate_{uart} * TonsUE_{uart} \tag{A.1}$$
$$+\sum_{u}\sum_{a}\sum_{r\in R_a}\sum_{t\in T_{uar}} LatePenPAX_u * DaysLate_{uart} * TPAX_{uart}$$
$$+\sum_{u}\sum_{i}\sum_{k}\left(NoGoPenUE_u * UENoGo_{uik} + NoGoPenPAX_u * PAXNoGo_{uik}\right)$$
$$+\sum_{a}\sum_{t} Preserve_{at} * NPlanes_{at}$$

The objective function minimizes the total weighted penalties incurred for late deliveries and non-deliveries. The model's secondary objective is to choose a feasible solution that maximizes unused aircraft.

F. CONSTRAINTS

$$\sum_{a\in A_{bulk}}\sum_{r\in R_{aik}}\sum_{t\in T_{uar}} TonsUE_{uart} + UENoGo_{uik} = MoveUE_{uik}, \tag{A.2}$$
$$\forall\ u,i,k\colon MoveUE_{uik} > 0$$

$$\sum_{a\in A_{out}}\sum_{r\in R_{aik}}\sum_{t\in T_{uar}} TonsUE_{uart} + UENoGo_{uik} \geq ProOut_u * MoveUE_{uik}, \tag{A.3}$$
$$\forall\ u,i,k\colon MoveUE_{uik} > 0$$

$$\sum_{a\in A_{ovr}}\sum_{r\in R_{aik}}\sum_{t\in T_{uar}} TonsUE_{uart} + UENoGo_{uik} \geq \tag{A.4}$$
$$\left(ProOver_u + ProOut_u\right) * MoveUE_{uik},\quad \forall\ u,i,k\colon MoveUE_{uik} > 0$$

$$\sum_{a} \sum_{r \in R_{ak}} \sum_{t \in T_{uar}} TPAX_{uart} + PaxNoGo_{uik} = MovePAX_{uik}, \qquad \text{(A.5)}$$

$$\forall\, u,i,k\text{: } MovePAX_{uik} > 0$$

$$\sum_{r} \sum_{r \in DR_i} X_{uart} + H_{ait} + Release_{ait} = \qquad \text{(A.6)}$$

$$H_{ai,t-1} + Allot_{ait} + \sum_{r \in R_{ai}} \sum_{t' + [CTime_{ar}] = t} Y_{art'}, \qquad \forall\, a,i,t$$

$$\sum_{r \in RR_k} Y_{art} + HP_{akt} = HP_{ak,t-1} + \sum_{u} \sum_{r \in R_{ak}} \sum_{\substack{t' \in T_{uar} \\ t' + [CTime_{ar}] = t}} X_{uart'}, \qquad \forall\, a,k,t \qquad \text{(A.7)}$$

$$\sum_{t'=1}^{t} \sum_{i} Allot_{ait} \leq \sum_{t'=1}^{t} Supply_{at}, \qquad \forall\, a,t \qquad \text{(A.8)}$$

$$NPlanes_{at} = \sum_{t'=1}^{t} \sum_{i} Allot_{ait'} - \sum_{t'=1}^{t} \sum_{i} Release_{ait'}, \qquad \forall\, a,t \qquad \text{(A.9)}$$

$$\sum_{r \in R_a} \sum_{t'=1}^{t} \sum_{u} K_{artt'} * X_{uart'} + \sum_{r \in R_a} \sum_{t'=1}^{t} K_{artt'} * Y_{art'} + \sum_{i} \sum_{t'=1}^{t} H_{ait'} \qquad \text{(A.10)}$$

$$+ \sum_{k} \sum_{t'=1}^{t} HP_{akt'} \leq \sum_{t'=1}^{t} NPlanes_{at}, \qquad \forall\, a,t$$

where

$$K_{artt'} = \begin{cases} t - t' + 1, & \text{if} \quad t' \leq t < t' + CTime_{ar} - 1 \\ CTime_{ar}, & \text{if} \quad t \geq t' + CTime_{ar} - 1 \end{cases}$$

$$TPAX_{uart} \leq MaxPAX_a * X_{uart}, \qquad \forall\, u,a,r,t\colon t \in T_{uar} \tag{A.11}$$

$$TonsUE_{uart} + PAXWt * TPAX_{uart} \leq MaxLoad_{ar} * X_{uart}, \qquad \forall\, u,a,r,t\colon t \in T_{uar} \tag{A.12}$$

$$\begin{aligned} &PAXSqFt_a * TPAX_{uart} + UESqFt_u * TonsUE_{uart} \leq \\ &\quad ACSqFt_a * LoadEff_a * X_{uart}, \qquad \forall\, u,a,r,t\colon t \in T_{uar} \end{aligned} \tag{A.13}$$

$$\begin{aligned} &\sum_{u} \sum_{r \in R_a} \sum_{t \in T_{uar}} FltTime_{ar} * X_{uart} + \sum_{r \in R_a} \sum_{t} FltTime_{ar} * Y_{art} \leq \\ &\quad \sum_{t} URate_a * NPlanes_{at}, \qquad \forall\, a \end{aligned} \tag{A.14}$$

$$\begin{aligned} &\sum_{u} \sum_{a} \sum_{r \in R_a} \sum_{\substack{t' \in T_{uar} \\ t' + [DTime_{abr}] = t}} \left(MOGReq_{ab} * GTime_{abr} / 24 \right) * X_{uart'} \\ &\quad + \sum_{a} \sum_{r \in R_a} \sum_{t' + [DTime_{abr}] = t} \left(MOGReq_{ab} * GTime_{abr} / 24 \right) * Y_{art'} \\ &\quad \leq MOGEff * MOGCap_b, \qquad \forall\, b,t \end{aligned} \tag{A.15}$$

A.2 Demand satisfaction constraints for all classes of cargo

A.3 Demand satisfaction constraints for out-sized cargo

A.4 Demand satisfaction constraint for over-sized cargo

A.5 Demand satisfaction for troops

A.6 Aircraft balance constraints at origin airfields

A.7 Aircraft balance constraints at destination airfields

A.8 Aircraft balance constraints for allocations to origins

A.9 Aircraft balance constraints accounting for allocations and releases

A.10 Cumulative aircraft balance constraints

A.11 Troop carriage capacity constraints

A.12 Maximum payload constraints

A.13 Cargo floor space constraints

A.14 Aircraft utilization constraints

A.15 Airfield MOG constraints

APPENDIX B. AIRCRAFT BREAK AND FIX RATES FOR 1994

	BREAKS	LANDINGS	BREAK RATE
C-5			
JAN	41	323	12.69
FEB	46	325	14.15
MAR	53	431	12.30
APR	64	499	12.83
MAY	51	440	11.59
JUN	83	384	21.61
JUL	60	443	13.54
AUG	53	419	12.65
SEP	37	346	10.69
OCT	56	369	15.18
NOV	67	462	14.50
DEC	36	366	9.84
C-141			
JAN	126	669	18.83
FEB	127	773	16.43
MAR	188	1085	17.33
APR	153	1169	13.09
MAY	143	1056	13.54
JUN	161	977	16.48
JUL	143	1128	12.68
AUG	117	1144	10.23
SEP	155	971	15.96
OCT	190	1045	18.18
NOV	163	1167	13.97
DEC	150	1014	14.79
C-17			
JAN	0	16	0.00
FEB	5	35	14.29
MAR	8	38	21.05
APR	3	60	5.00
MAY	7	69	10.14
JUN	3	54	5.56
JUL	4	71	5.63
AUG	4	94	4.26
SEP	0	83	0.00
OCT	6	127	4.72
NOV	10	106	9.43
DEC	8	89	8.99

	FIX 0-4 HRS	FIX RATE 0-4 HRS	FIX 4-8 HRS	FIX RATE 4-8 HRS	FIX 8-12 HRS	FIX RATE 8-12 HRS	FIX 12-16 HRS	FIX RATE 12-16 HRS
C-5								
JAN	8	19.5	8	19.5	6	14.6	4	9.8
FEB	8	17.4	6	13.0	6	13.0	6	13.0
MAR	14	26.4	17	32.1	8	15.1	3	5.7
APR	16	25.0	18	28.1	5	7.8	11	17.2
MAY	15	29.4	11	21.6	3	5.9	5	9.8
JUN	11	13.3	16	19.3	11	13.3	9	10.8
JUL	15	25.0	15	25.0	6	10.0	7	11.7
AUG	14	26.4	9	17.0	7	13.2	6	11.3
SEP	10	27.0	10	27.0	4	10.8	4	10.8
OCT	17	30.4	11	19.6	7	12.5	5	8.9
NOV	15	22.4	11	16.4	8	11.9	11	16.4
DEC	5	13.9	9	25.0	8	22.2	5	13.9
C-141								
JAN	50	39.7	27	21.4	18	14.3	11	8.7
FEB	40	31.5	27	21.3	37	29.1	7	5.5
MAR	77	41.0	42	22.3	29	15.4	15	8.0
APR	65	42.5	49	32.0	18	11.8	13	8.5
MAY	65	45.5	43	30.1	15	10.5	10	7.0
JUN	74	46.0	44	27.3	20	12.4	9	5.6
JUL	60	42.0	27	18.9	16	11.2	15	10.5
AUG	68	58.1	13	11.1	14	12.0	3	2.6
SEP	65	41.9	31	20.0	19	12.3	14	9.0
OCT	63	33.2	52	27.4	33	17.4	15	7.9
NOV	61	37.4	25	15.3	17	10.4	15	9.2
DEC	50	33.3	32	21.3	22	14.7	20	13.3
C-17								
JAN	0	0.0	0	0.0	0	0	0	0
FEB	1	20.0	0	0.0	1	20	0	0
MAR	3	37.5	0	0.0	0	0	2	25
APR	2	66.7	0	0.0	0	0	0	0
MAY	2	28.6	2	28.6	0	0	0	0
JUN	1	33.3	1	33.3	0	0	0	0
JUL	3	75.0	0	0.0	0	0	0	0
AUG	0	0.0	1	25.0	0	0	0	0
SEP	0	0.0	0	0.0	0	0	0	0
OCT	4	66.7	0	0.0	0	0	0	0
NOV	4	40.0	1	10.0	1	10	0	0
DEC	3	37.5	2	25.0	0	0	0	0

	FIX 16-24 HRS	FIX RATE 16-24 HRS	FIX 24-48 HRS	FIX RATE 24-48 HRS	FIX 48-72 HRS	FIX RATE 48-72 HRS	FIX >72 HRS	FIX RATE >72 HRS
C-5								
JAN	2	4.9	10	24.4	1	2.4	5	12.2
FEB	8	17.4	8	17.4	1	2.2	3	6.5
MAR	7	13.2	3	5.7	1	1.9	0	0.0
APR	5	7.8	7	10.9	2	3.1	0	0.0
MAY	4	7.8	7	13.7	4	7.8	2	3.9
JUN	6	7.2	6	7.2	5	6.0	3	3.6
JUL	2	3.3	10	16.7	3	5.0	2	3.3
AUG	9	17.0	7	13.2	0	0.0	1	1.9
SEP	2	5.4	6	16.2	1	2.7	0	0.0
OCT	6	10.7	8	14.3	1	1.8	1	1.8
NOV	6	9.0	13	19.4	0	0.0	3	4.5
DEC	4	11.1	5	13.9	0	0.0	0	0.0
C-141								
JAN	9	7.1	5	4.0	3	2.4	1	0.8
FEB	7	5.5	4	3.1	1	0.8	4	3.1
MAR	12	6.4	11	5.9	2	1.1	0	0.0
APR	6	3.9	1	0.7	1	0.7	0	0.0
MAY	5	3.5	5	3.5	0	0.0	0	0.0
JUN	5	3.1	5	3.1	1	0.6	2	1.2
JUL	14	9.8	7	4.9	4	2.8	0	0.0
AUG	11	9.4	4	3.4	2	1.7	2	1.7
SEP	15	9.7	7	4.5	2	1.3	2	1.3
OCT	6	3.2	13	6.8	0	0.0	0	0.0
NOV	13	8.0	17	10.4	4	2.5	1	0.6
DEC	8	5.3	8	5.3	6	4.0	4	2.7
C-17								
JAN	0	0.0	0	0.0	0	0.0	0	0.0
FEB	1	20.0	2	40.0	0	0.0	0	0.0
MAR	1	12.5	1	12.5	1	12.5	0	0.0
APR	0	0.0	0	0.0	0	0.0	1	33.3
MAY	0	0.0	0	0.0	0	0.0	0	0.0
JUN	1	33.3	0	0.0	0	0.0	0	0.0
JUL	0	0.0	1	25.0	0	0.0	0	0.0
AUG	1	25.0	0	0.0	1	25.0	1	25.0
SEP	0	0.0	0	0.0	1	0.0	0	0.0
OCT	1	16.7	1	16.7	1	16.7	0	0.0
NOV	1	10.0	3	30.0	1	10.0	0	0.0
DEC	1	12.5	1	12.5	1	12.5	0	0.0

APPENDIX C. BENDERS DECOMPOSITION CALCULATIONS

This appendix calculates the cut gradient and intercept expressions used in Benders algorithm for solving the stochastic optimization model. As shown in Chapter IV, Section C, the algorithm will add the following cuts at each iteration

$$-G_{bt}^{i}x+\theta_{bt}\geq g_{bt}^{i}\quad\forall\ b,t. \tag{C.1}$$

Since x represents the first-stage decision variables used in the Throughput II model, constraint (C.1) is equivalent to

$$\sum_{u}\sum_{a}\sum_{r}\sum_{t'}G_{abt}^{i}*X_{uart'}+\sum_{a}\sum_{r}\sum_{t'}G_{abt}^{i}*Y_{art'}+\theta_{bt}\geq g_{bt}^{i}\quad\forall\ b,t. \tag{C.2}$$

To calculate expressions for G_{abt}^{i} and g_{bt}^{i}, the first step is to take the dual of the recourse function. In this case, the primal is

$$h_{bt}(x,\omega)=\min_{R_{bt}^{\omega}}\ MOGPEN_{bt}*R_{bt}^{\omega} \tag{C.3}$$

subject to

(1) $\sum_{u}\sum_{a}\sum_{r}\sum_{t'}\left[MOG\,\mathrm{Re}q_{ab}*\left(GTime_{abr}+MRTime_{abt}^{\omega}\right)/24\right]*X_{uart'}$

$+\sum_{a}\sum_{r}\sum_{t'}\left[MOG\,\mathrm{Re}q_{ab}*\left(GTime_{abr}+MRTime_{abt}^{\omega}\right)/24\right]*Y_{art'}$

$-R_{bt}^{\omega}\leq MOGCap_{b}$

(2) $R_{bt}^{\omega}\geq 0$.

The dual of the recourse function is

$$h_{bt}(x,\omega) = \max_{\lambda_{bt}^{\omega}} \tag{C.4}$$

$$\lambda_{bt}^{\omega}*\left\{\begin{array}{l}-MOGCAP_{b}\\+\sum_{u}\sum_{a}\sum_{r}\sum_{t'}\left[MOG\,\mathrm{Re}q_{ab}*\left(GTime_{abr}+MRTime_{abt}^{\omega}\right)/24\right]*X_{uart'}\\+\sum_{a}\sum_{r}\sum_{t'}\left[MOG\,\mathrm{Re}q_{ab}*\left(GTime_{abr}+MRTime_{abt}^{\omega}\right)/24\right]*Y_{art'}\end{array}\right\}$$

subject to

(1) $\lambda_{bt}^{\omega}\leq MOGPEN_{bt}$

(2) $\lambda_{bt}^{\omega}\geq 0$.

From the L-Shaped Algorithm for stochastic programming (Morton,1994), we know $G^i = E_\omega \pi^\omega B^\omega$ and $g^i = E_\omega \pi^\omega d^\omega$ where π^ω denotes the optimal dual vector. Application to this model yields the following expression the cut gradient

$$G^i_{bt} = E_\omega \pi^\omega_{bt} B^\omega = E_\omega \lambda^\omega_{bt} MOG\,\mathrm{Re}q_{ab} * \left(GTime_{abr} + MRTime^\omega_{abt}\right)/24 \tag{C.5}$$

and the following for the cut intercept

$$g^i_{bt} = E_\omega \pi^\omega_{bt} d^\omega = E_\omega \lambda^\omega_{bt}\left(-MOGCap_b\right). \tag{C.6}$$

The solution to the dual problem will depend on two possible conditions that can exist. In the first case, if there is no MOG violation at airfield *b*, time period *t* and scenario *ω*, the corresponding recourse variable R^ω_{bt} will equal zero. In this case, the corresponding dual variable λ^ω_{bt} is equal to zero. On the other hand, if there is a MOG violation at airfield *b*, time period *t* and scenario *ω*, the corresponding recourse variable R^ω_{bt} will be greater than zero. In this case, the corresponding dual variable λ^ω_{bt} is equal to $MOGPEN_{bt}$. In summary, we have

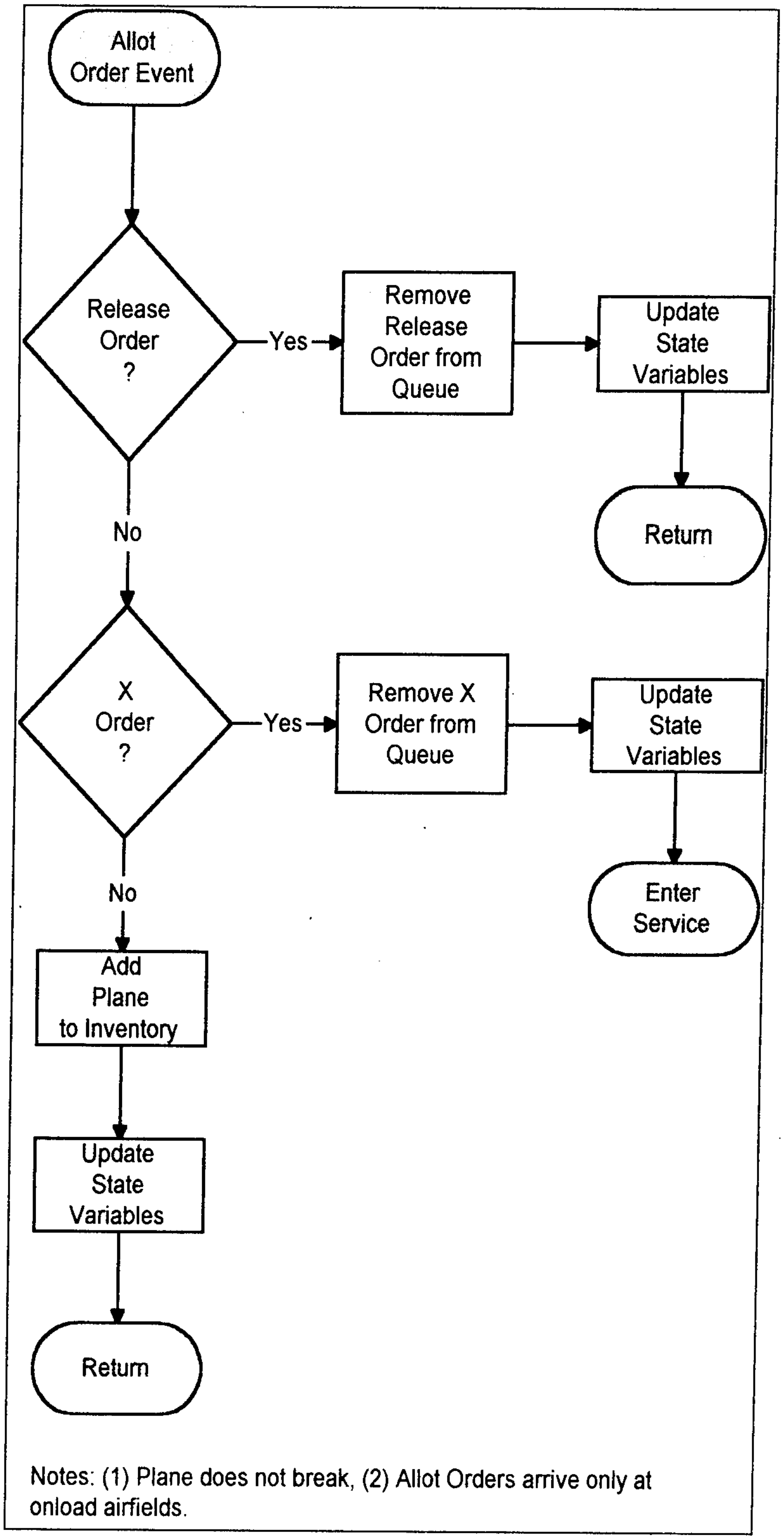
$$\text{Condition I:} \quad R^\omega_{bt} = 0 \qquad \lambda^\omega_{bt} = 0 \tag{C.7}$$

$$\text{Condition II:} \quad R^\omega_{bt} > 0 \qquad \lambda^\omega_{bt} = MOGPEN_{bt}. \tag{C.8}$$

APPENDIX D. SIMULATION EVENT DIAGRAMS

Appendix D contains the event diagrams for the simulation model. The following abbreviations are used:

MC	mission capable, aircraft not broken
MRT	modified repair time realization
RT	repair time realization.


Allot Order Event
Release Order ?
Yes
Remove Release Order from Queue
Update State Variables
Return
No
X Order ?
Yes
Remove X Order from Queue
Update State Variables
Enter Service
No
Add Plane to Inventory
Update State Variables
Return

Notes: (1) Plane does not break, (2) Allot Orders arrive only at onload airfields.

Figure D-1: Allot Order Event Diagram.

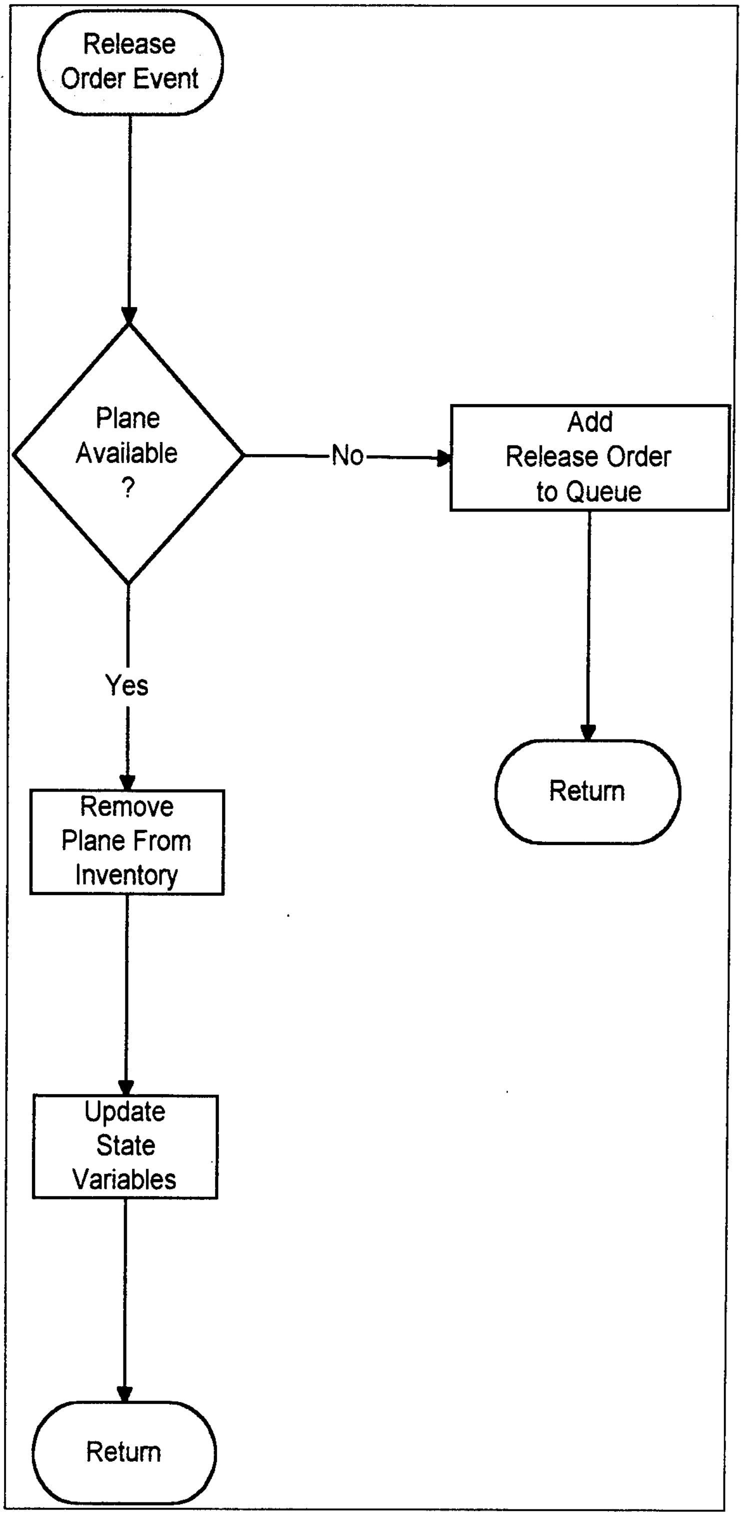

Release Order Event
Plane Available ?
No
Add Release Order to Queue
Return
Yes
Remove Plane From Inventory
Update State Variables
Return

Figure D-2: Release Order Event Diagram.

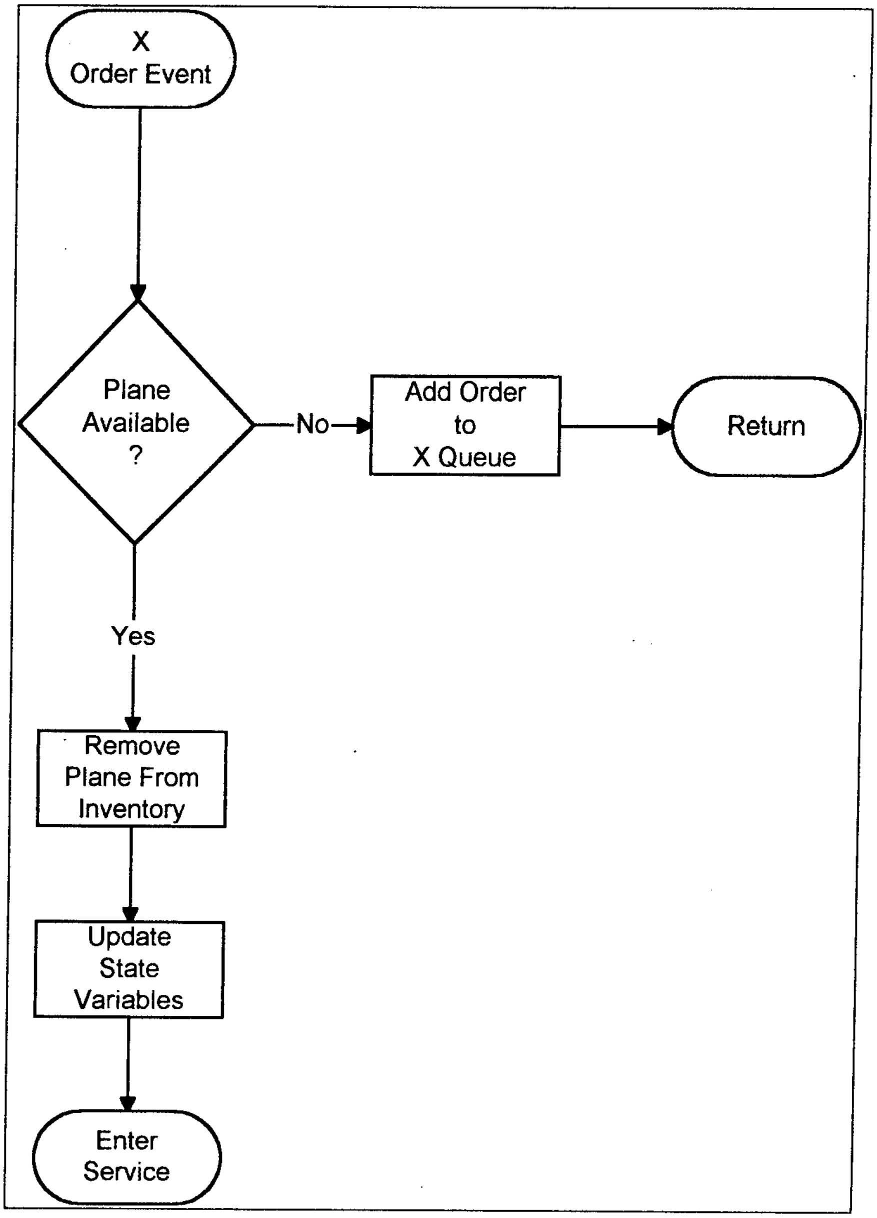

X
Order Event
Plane
Available
?
No
Add Order
to
X Queue
Return
Yes
Remove
Plane From
Inventory
Update
State
Variables
Enter
Service

Figure D-3: X Order Event Diagram.

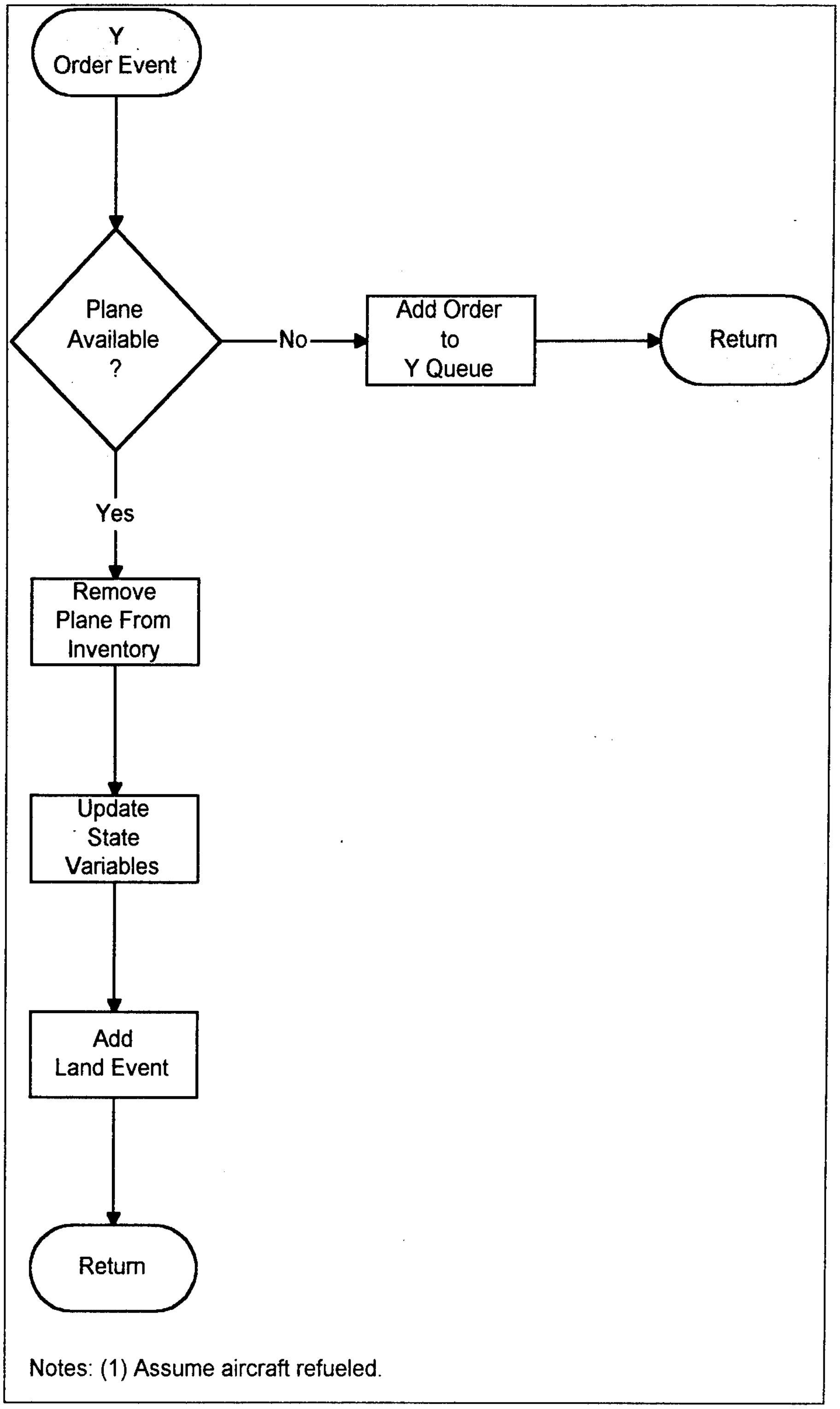
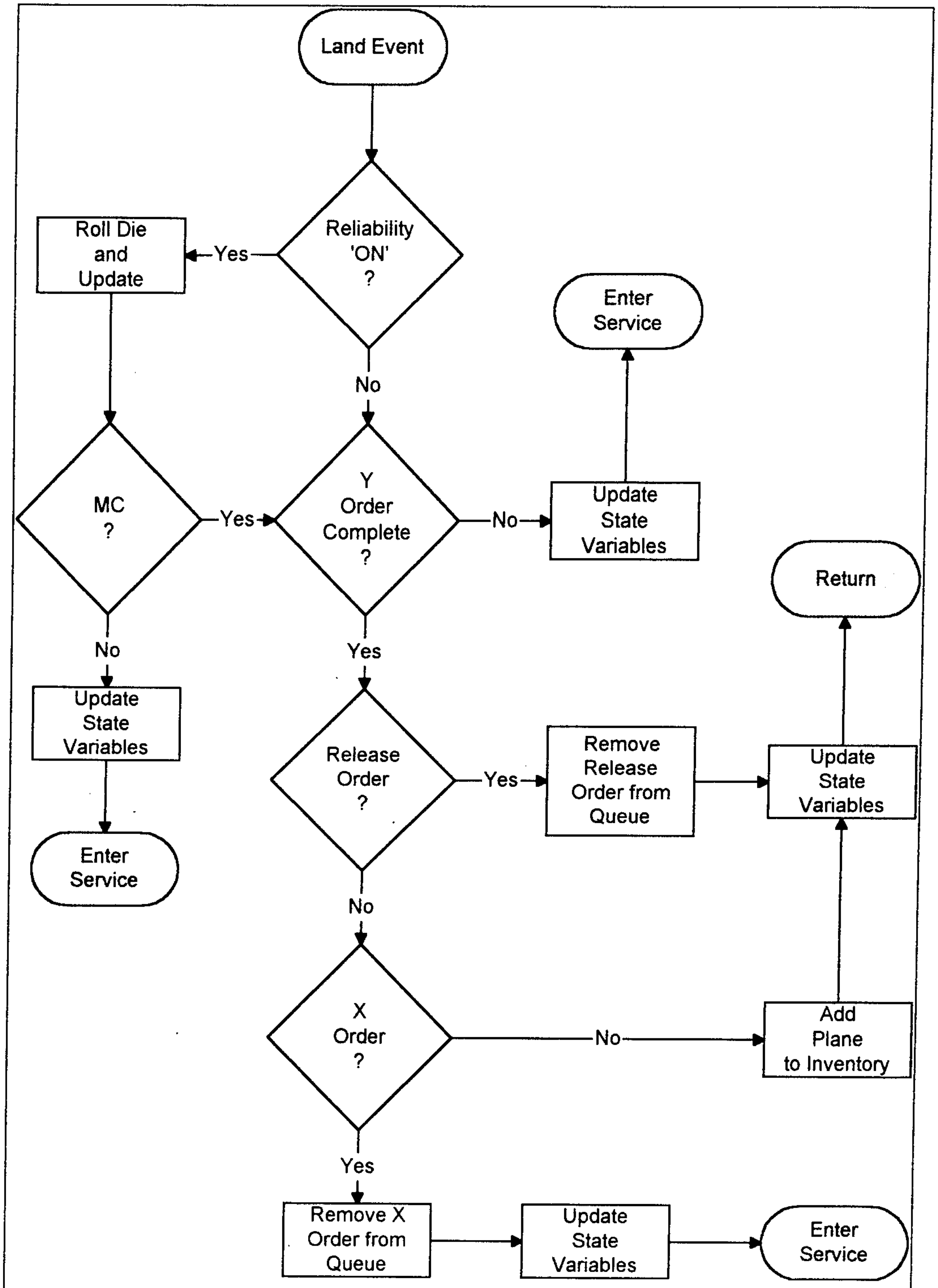

Y
Order Event
Plane
Available
?
No
Add Order
to
Y Queue
Return
Yes
Remove
Plane From
Inventory
Update
State
Variables
Add
Land Event
Return
Notes: (1) Assume aircraft refueled.

Figure D-4: Y Order Event Diagram.


Land Event
Reliability 'ON' ?
Yes
Roll Die and Update
No
Enter Service
MC ?
Yes
Y Order Complete ?
No
Update State Variables
Return
No
Update State Variables
Yes
Enter Service
Release Order ?
Yes
Remove Release Order from Queue
Update State Variables
No
X Order ?
No
Add Plane to Inventory
Yes
Remove X Order from Queue
Update State Variables
Enter Service

Notes: (1) If an aircraft completes a Y order and breaks, the simulation first repairs the aircraft, and then (if there exist an X order) assigns an X order to the aircraft.

Figure D-5: Land Event Diagram.

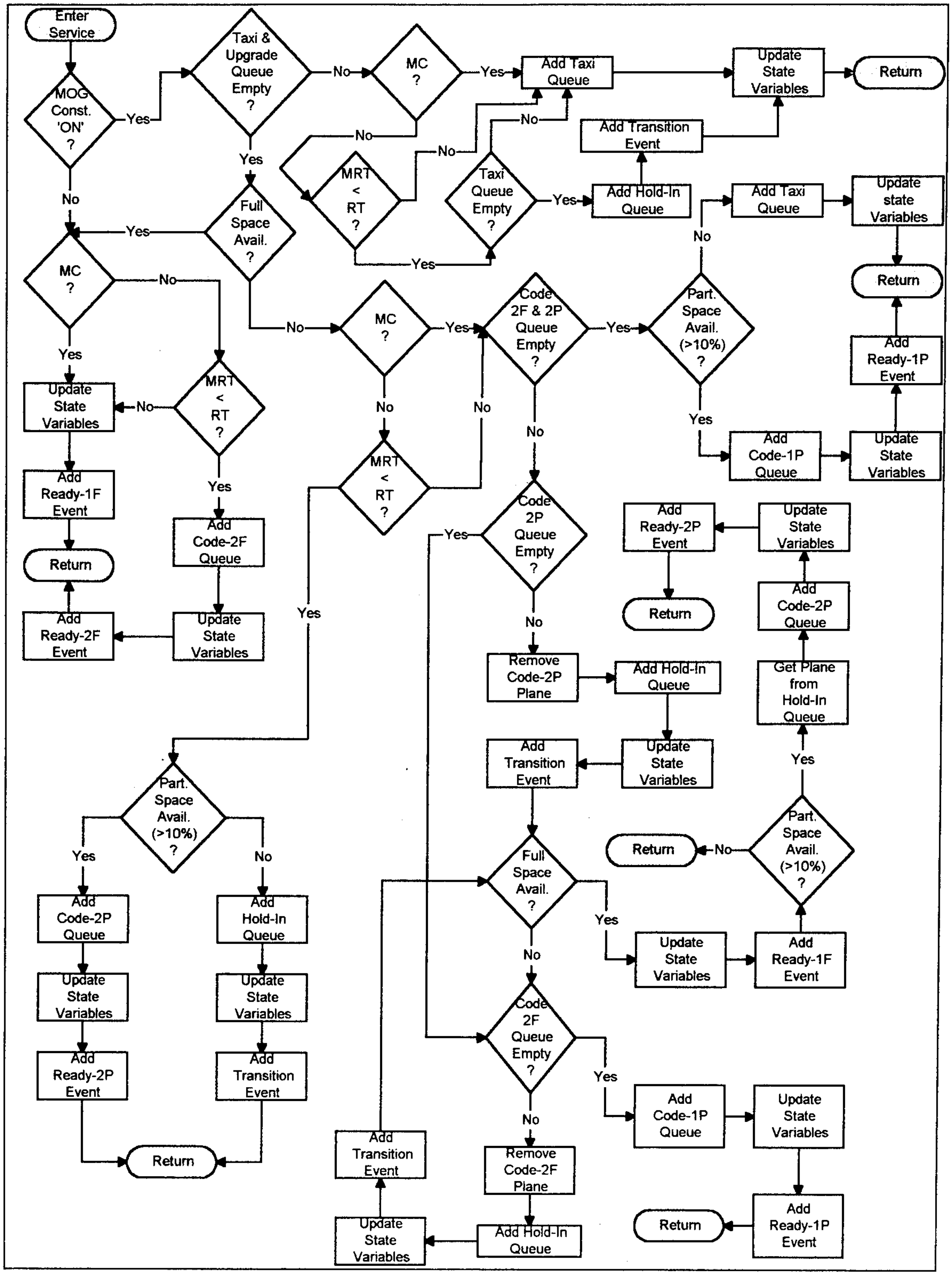
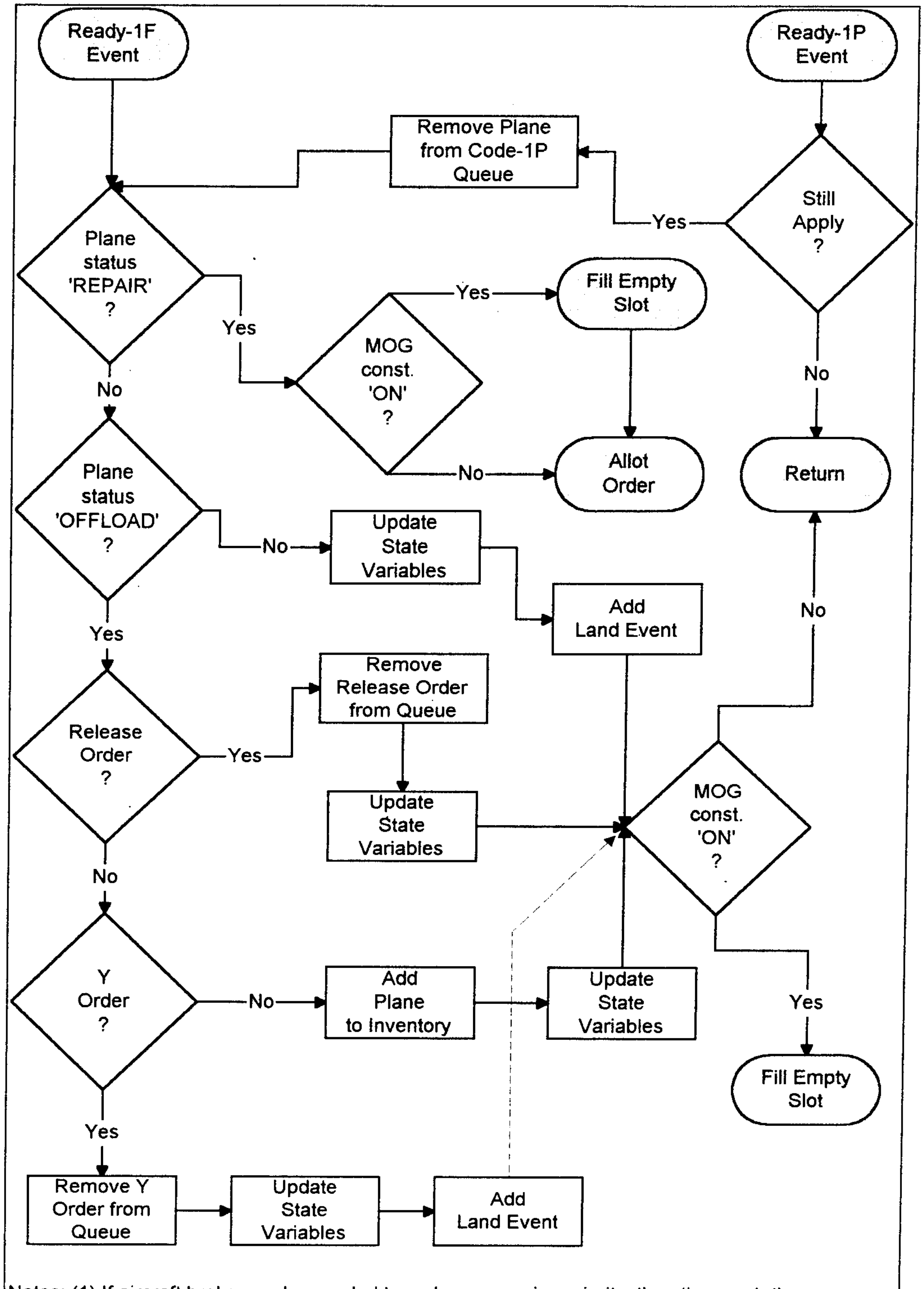

Enter Service
MOG Const. 'ON' ?
Yes
No
Taxi & Upgrade Queue Empty ?
MC ?
Add Taxi Queue
Update State Variables
Return
Add Transition Event
MRT < RT ?
Taxi Queue Empty ?
Add Hold-In Queue
Add Taxi Queue
Update state Variables
Full Space Avail. ?
MC ?
Code 2F & 2P Queue Empty ?
Part. Space Avail. (>10%) ?
Add Ready-1P Event
Add Code-1P Queue
Update State Variables
Add Ready-1F Event
Add Code-2F Queue
Add Ready-2F Event
Code 2P Queue Empty ?
Add Ready-2P Event
Add Code-2P Queue
Remove Code-2P Plane
Get Plane from Hold-In Queue
Add Transition Event
Code 2F Queue Empty ?
Remove Code-2F Plane
Add Hold-In Queue
Add Code-2P Queue
Add Ready-2P Event
Add Hold-In Queue
Add Transition Event
Return

Figure D-6: Enter Service Diagram.


Ready-1F Event
Ready-1P Event
Remove Plane from Code-1P Queue
Still Apply ?
Yes
No
Plane status 'REPAIR' ?
Yes
No
MOG const. 'ON' ?
Yes
No
Fill Empty Slot
Allot Order
Return
Plane status 'OFFLOAD' ?
No
Yes
Update State Variables
Add Land Event
No
Release Order ?
Yes
No
Remove Release Order from Queue
Update State Variables
MOG const. 'ON' ?
Yes
Fill Empty Slot
Y Order ?
No
Yes
Add Plane to Inventory
Update State Variables
Remove Y Order from Queue
Update State Variables
Add Land Event

Notes: (1) If aircraft broken and upgraded to code-one service priority, then the repair time requirement has been satisfied.

Figure D-7: Ready-1F and Ready-1P Event Diagrams.

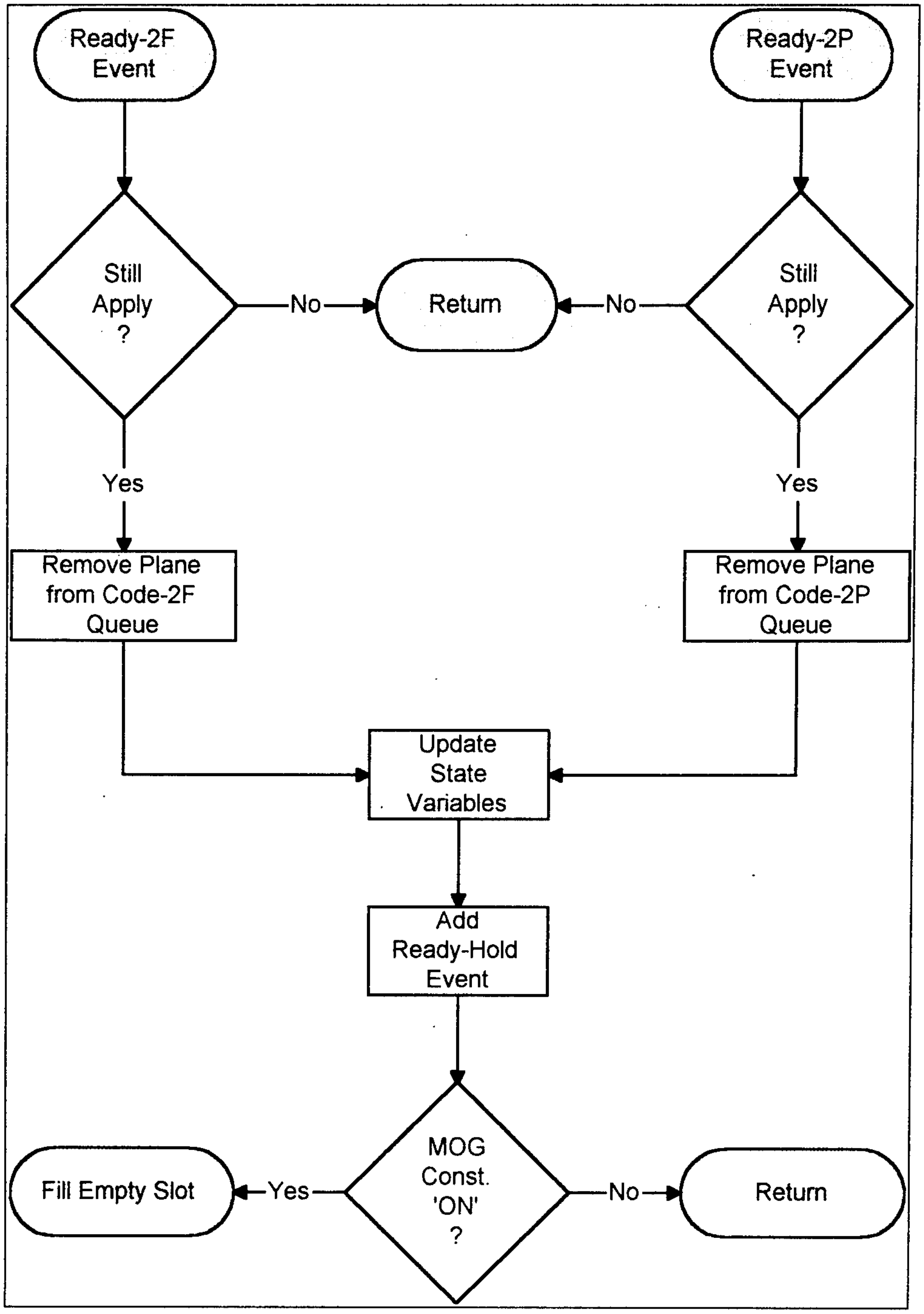
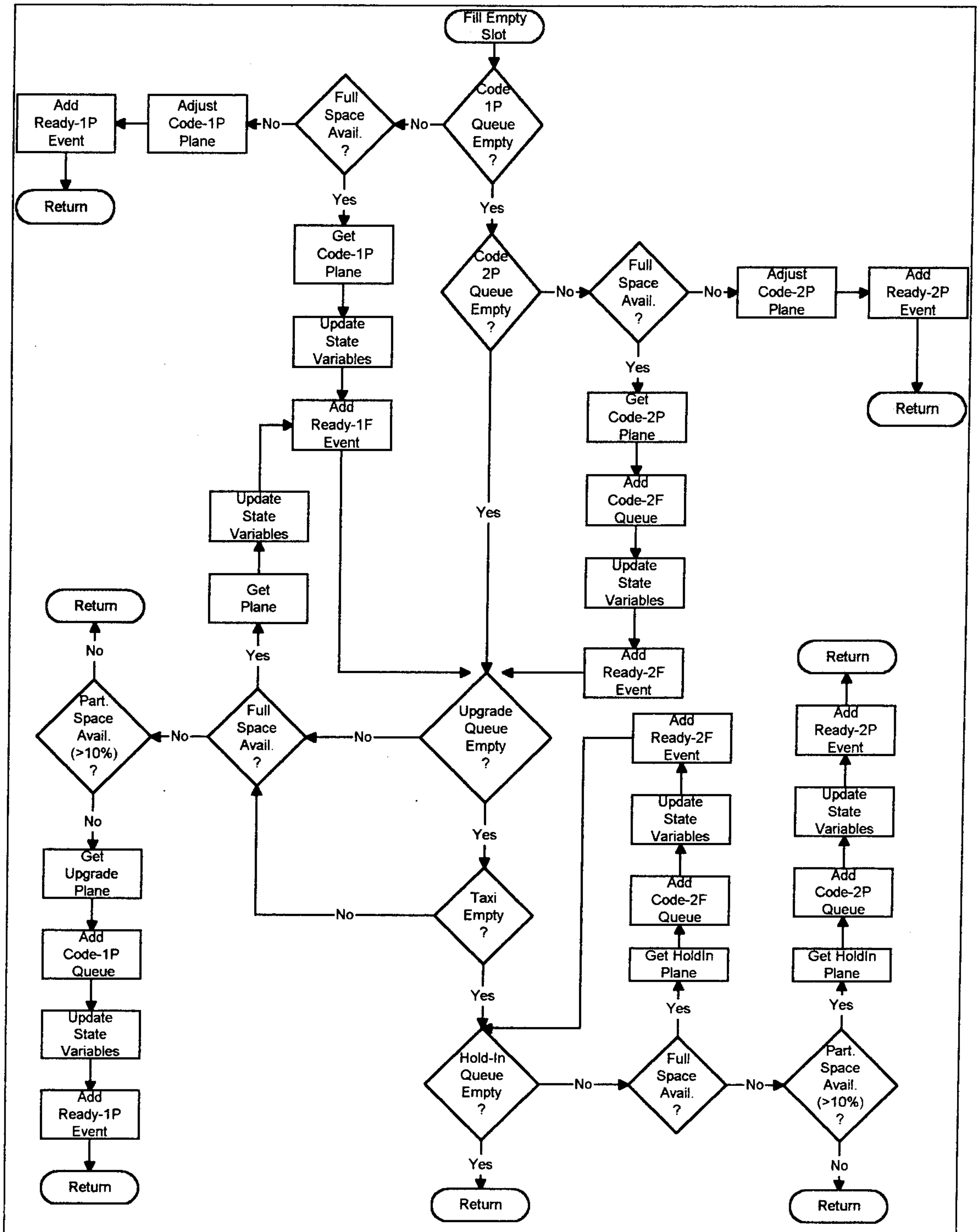
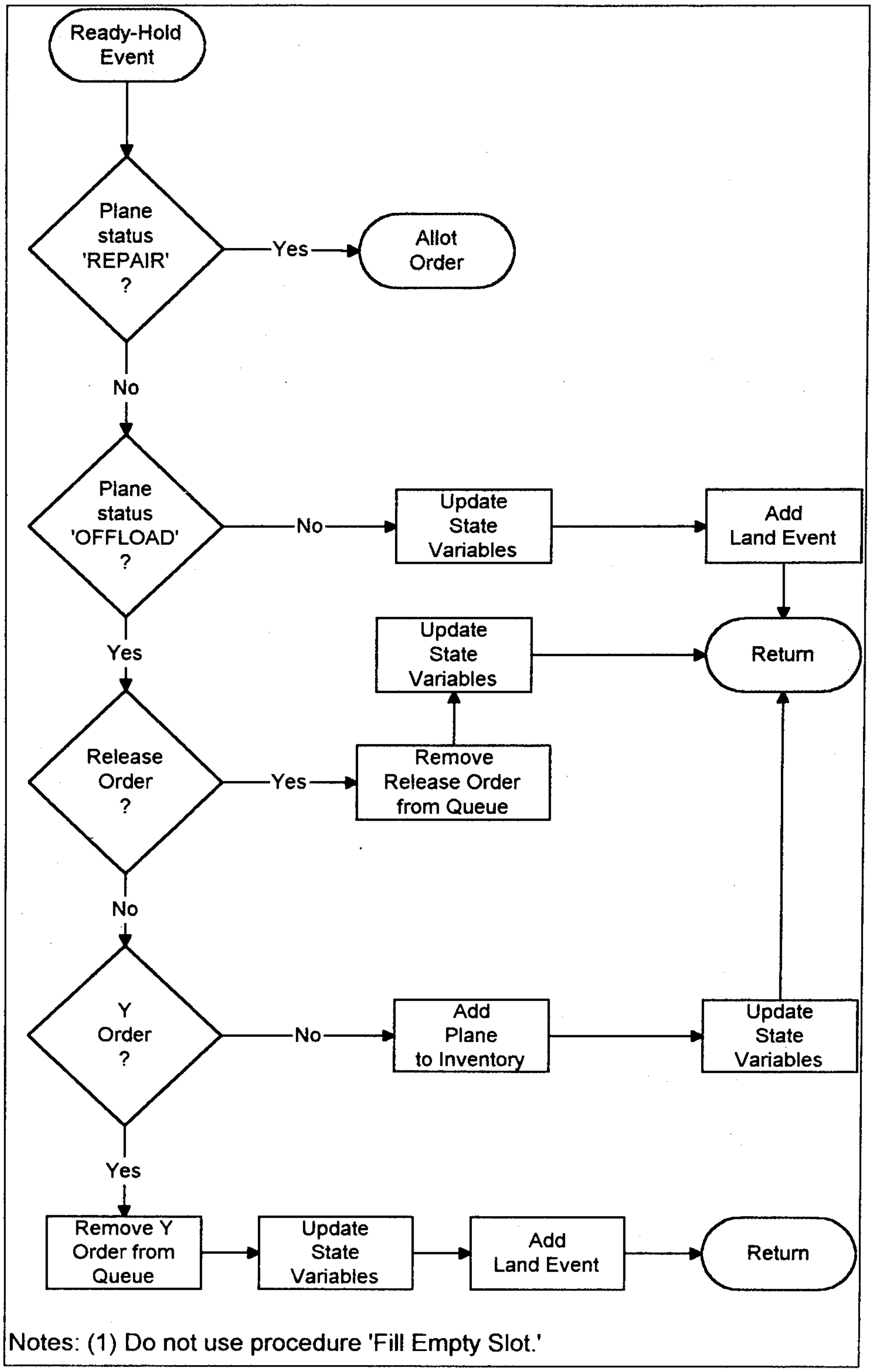

Ready-2F Event
Ready-2P Event
Still Apply ?
No
Return
No
Still Apply ?
Yes
Yes
Remove Plane from Code-2F Queue
Remove Plane from Code-2P Queue
Update State Variables
Add Ready-Hold Event
Fill Empty Slot
Yes
MOG Const. 'ON' ?
No
Return

Figure D-8: Ready-2F and Ready-2P Event Diagrams.


Fill Empty Slot
Code 1P Queue Empty ?
No
Full Space Avail. ?
No
Adjust Code-1P Plane
Add Ready-1P Event
Return
Yes
Get Code-1P Plane
Update State Variables
Add Ready-1F Event
Yes
Code 2P Queue Empty ?
No
Full Space Avail. ?
No
Adjust Code-2P Plane
Add Ready-2P Event
Return
Yes
Get Code-2P Plane
Add Code-2F Queue
Update State Variables
Add Ready-2F Event
Yes
Upgrade Queue Empty ?
No
Full Space Avail. ?
Yes
Get Plane
Update State Variables
No
Part. Space Avail. (>10%) ?
No
Return
No
Get Upgrade Plane
Add Code-1P Queue
Update State Variables
Add Ready-1P Event
Return
Yes
Taxi Empty ?
No
Yes
Hold-In Queue Empty ?
Yes
Return
No
Full Space Avail. ?
Yes
Get HoldIn Plane
Add Code-2F Queue
Update State Variables
Add Ready-2F Event
No
Part. Space Avail. (>10%) ?
Yes
Get HoldIn Plane
Add Code-2P Queue
Update State Variables
Add Ready-2P Event
Return
No
Return

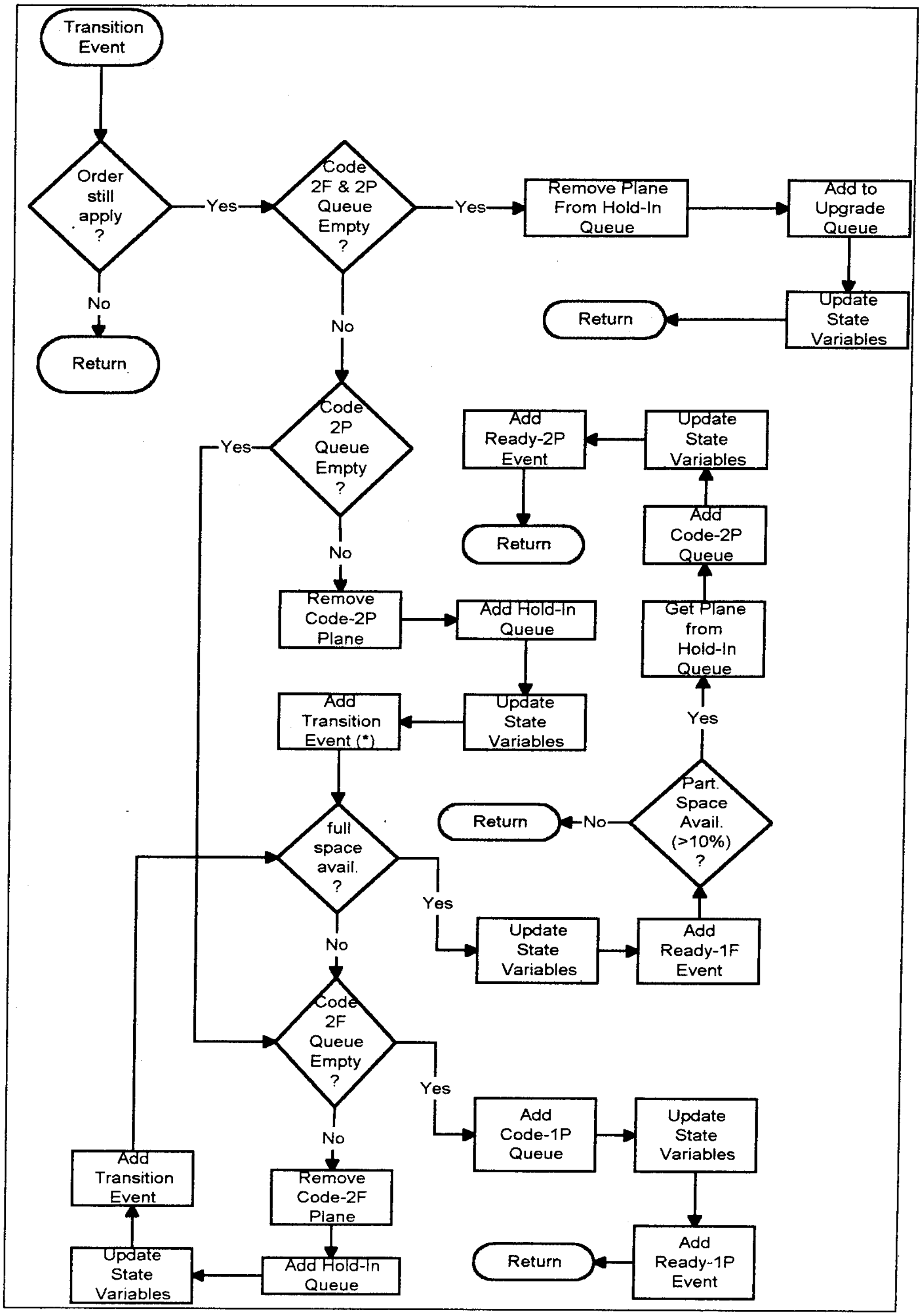
Notes: (1) if the next aircraft in the Taxi Queue has a code-two service priority, the aircraft is moved into the Hold-In Queue.

Figure D-9: Fill Empty Slot Diagram.



Ready-Hold Event
Plane status 'REPAIR' ?
Yes
Allot Order
No
Plane status 'OFFLOAD' ?
No
Update State Variables
Add Land Event
Yes
Update State Variables
Return
Release Order ?
Yes
Remove Release Order from Queue
No
Y Order ?
No
Add Plane to Inventory
Update State Variables
Yes
Remove Y Order from Queue
Update State Variables
Add Land Event
Return
Notes: (1) Do not use procedure 'Fill Empty Slot.'

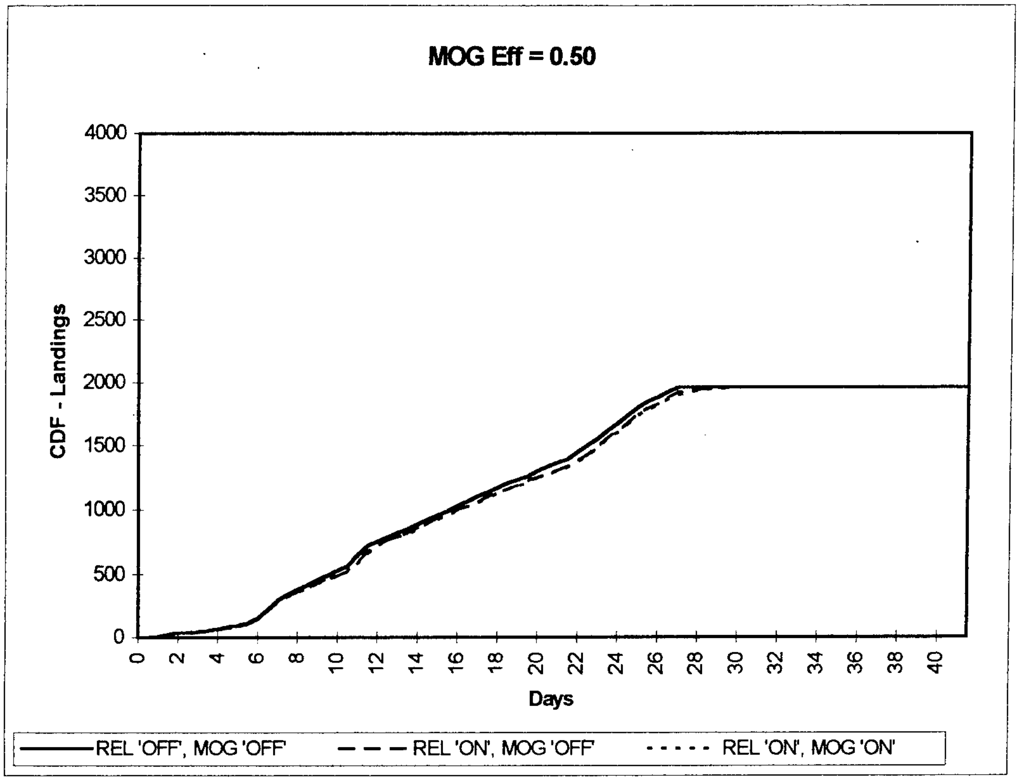
Figure D-10: Ready-Hold Event Diagram.

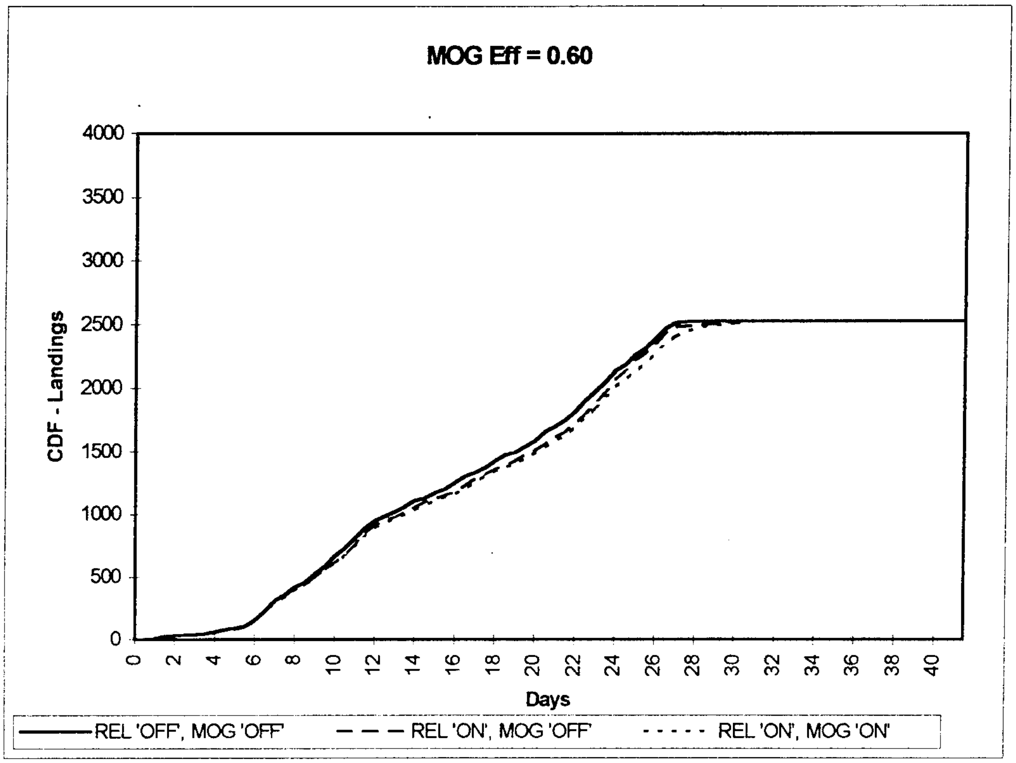


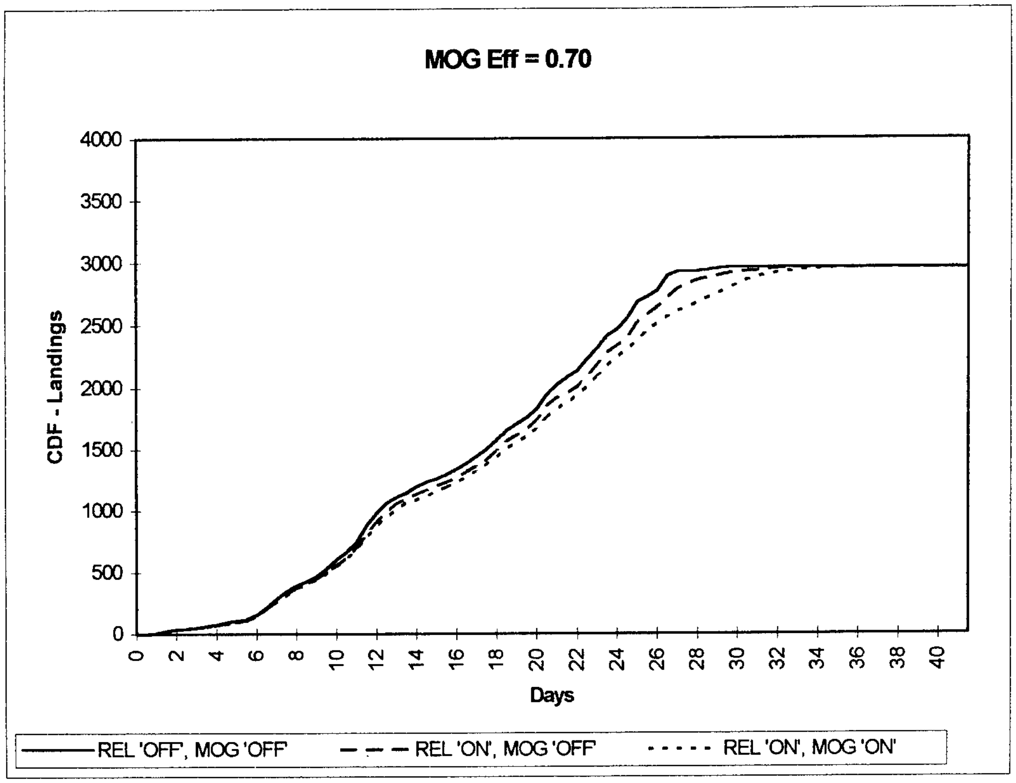
Transition Event
Order still apply ?
Yes
No
Return
Code 2F & 2P Queue Empty ?
Yes
Remove Plane From Hold-In Queue
Add to Upgrade Queue
Update State Variables
Return
No
Code 2P Queue Empty ?
Yes
No
Add Ready-2P Event
Update State Variables
Return
Add Code-2P Queue
Remove Code-2P Plane
Add Hold-In Queue
Get Plane from Hold-In Queue
Add Transition Event (*)
Update State Variables
Yes
Part. Space Avail. (>10%) ?
Return
No
full space avail. ?
Yes
Update State Variables
Add Ready-1F Event
No
Code 2F Queue Empty ?
Yes
No
Add Code-1P Queue
Update State Variables
Add Transition Event
Remove Code-2F Plane
Update State Variables
Add Hold-In Queue
Return
Add Ready-1P Event

Figure D-11: Transition Event Diagram.

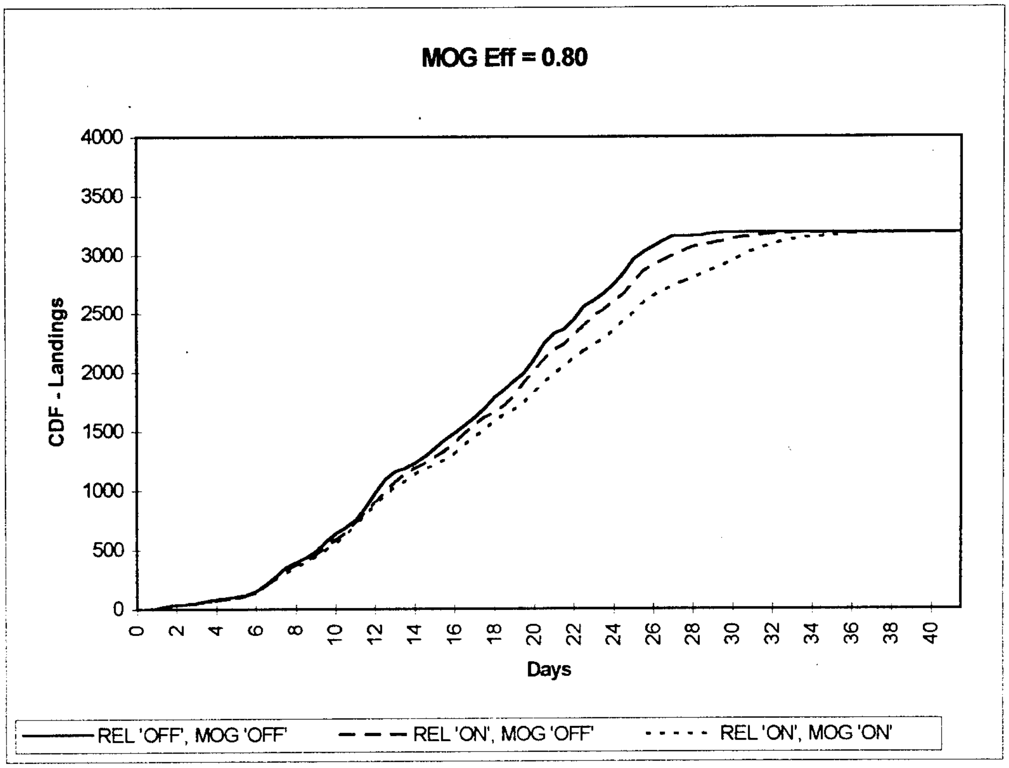
APPENDIX E. MOG EFFICIENCY FACTOR - LANDINGS


MOG Eff = 0.50
CDF - Landings
4000
3500
3000
2500
2000
1500
1000
500
0
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
Days
REL 'OFF', MOG 'OFF'
REL 'ON', MOG 'OFF'
REL 'ON', MOG 'ON'

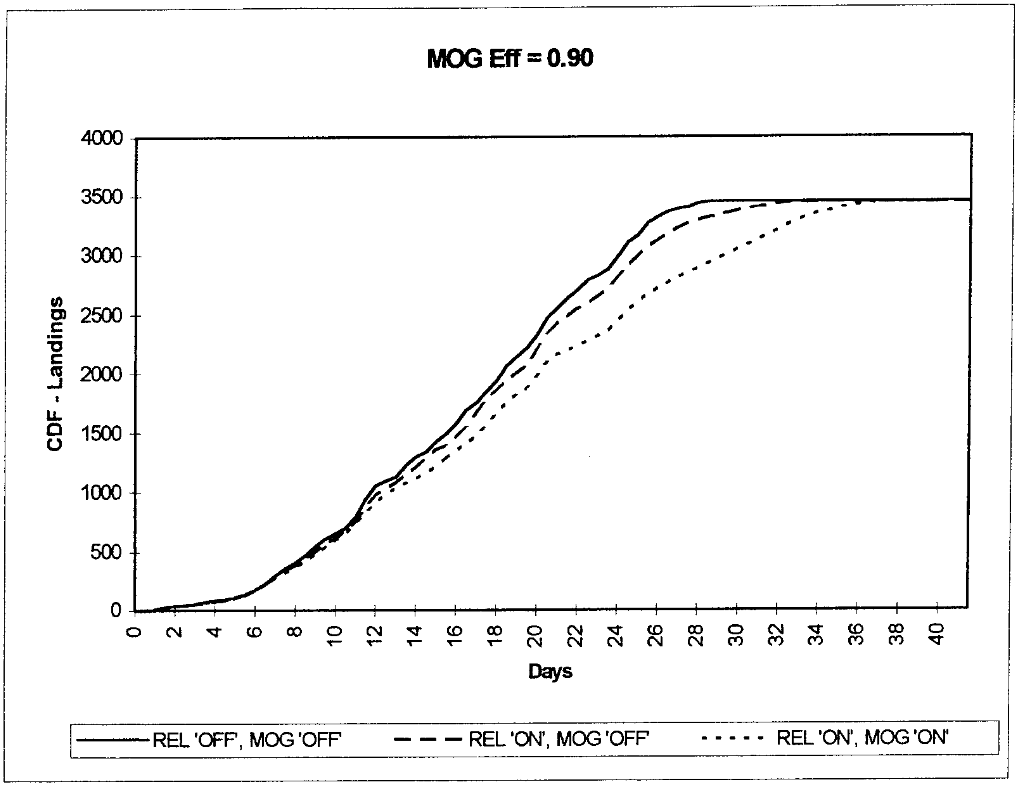

MOG Eff = 0.60
CDF - Landings
4000
3500
3000
2500
2000
1500
1000
500
0
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
Days
REL 'OFF', MOG 'OFF'
REL 'ON', MOG 'OFF'
REL 'ON', MOG 'ON'

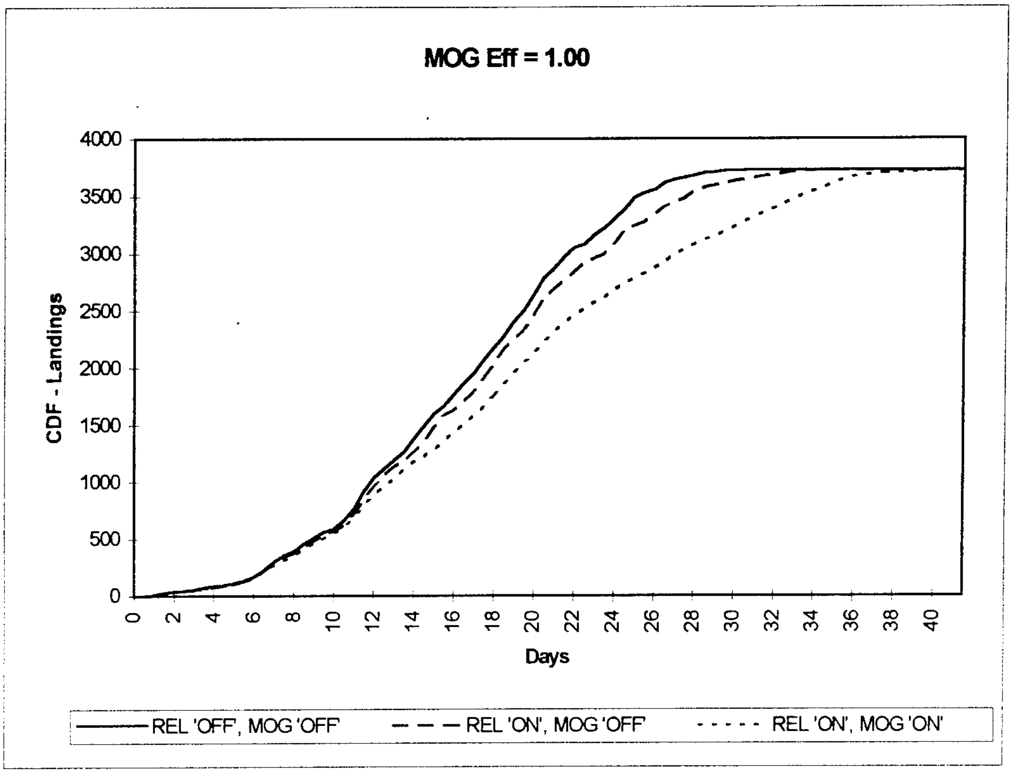


MOG Eff = 0.70

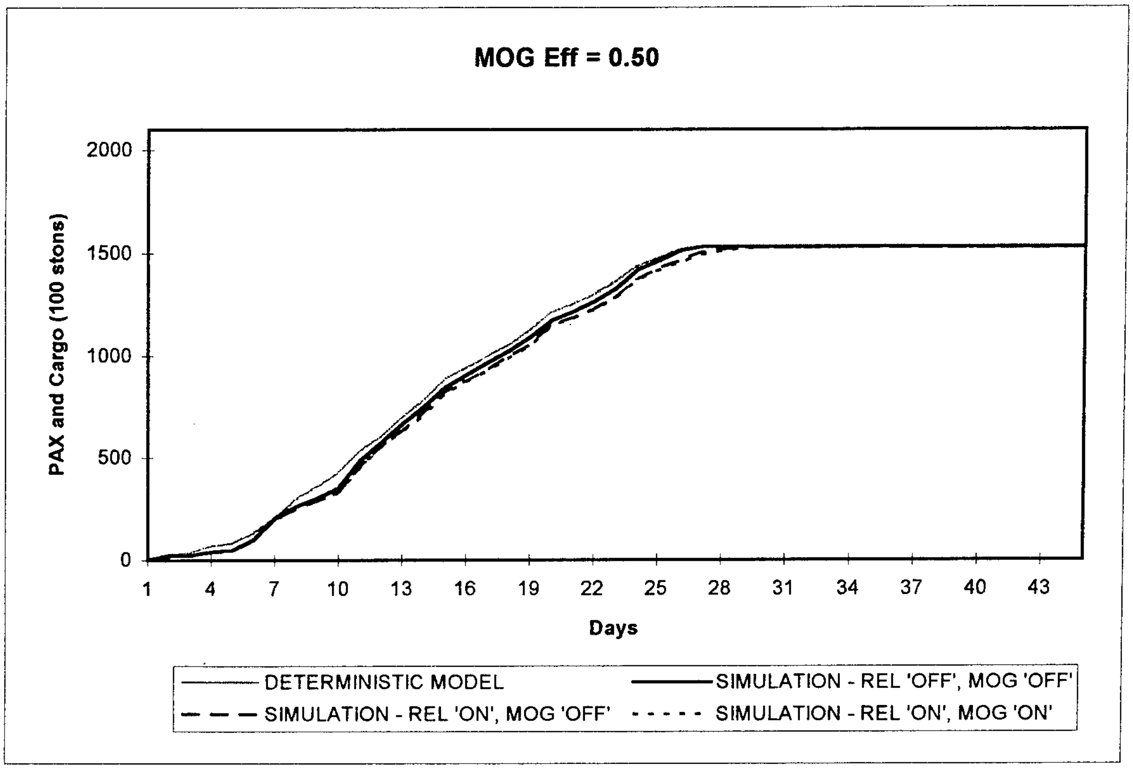


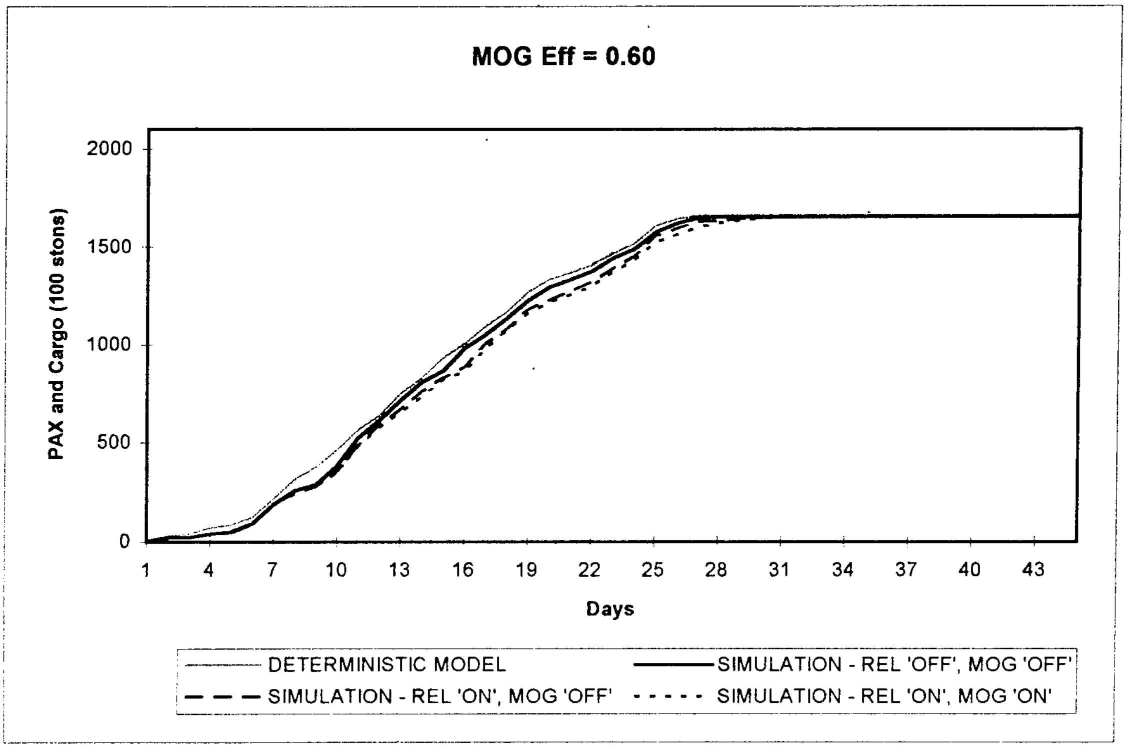
MOG Eff = 0.80

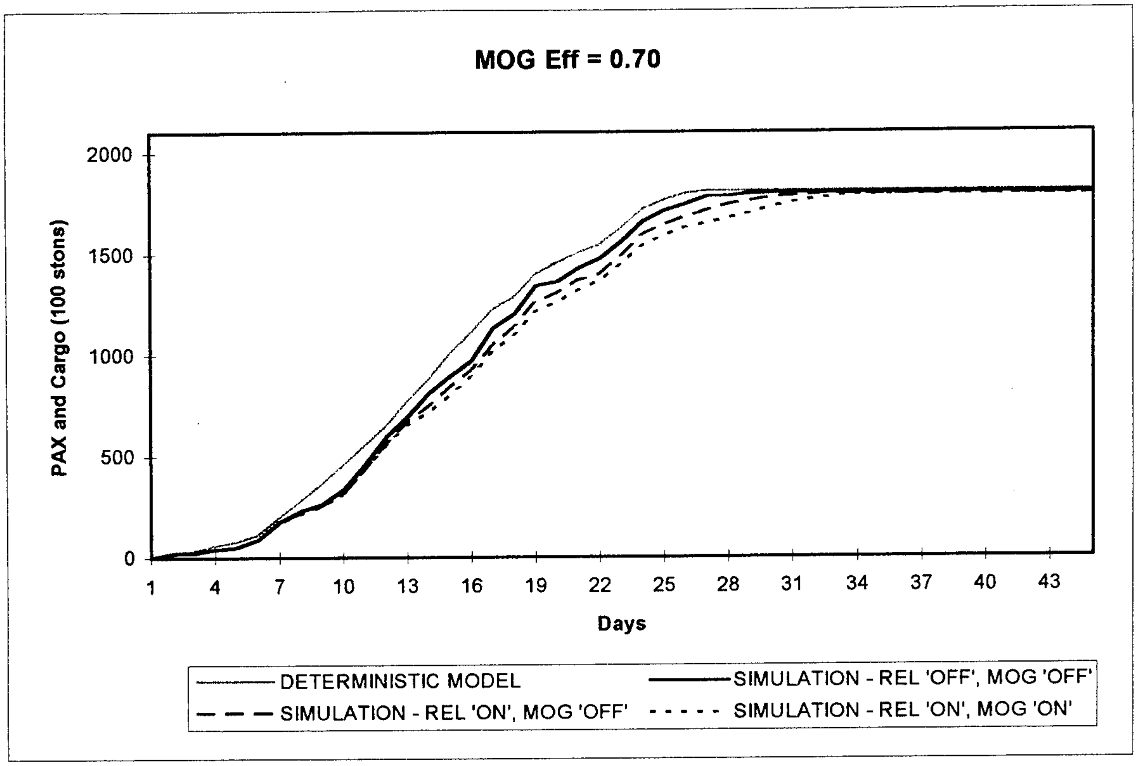

MOG Eff = 0.90
CDF - Landings
4000
3500
3000
2500
2000
1500
1000
500
0
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
Days
REL 'OFF', MOG 'OFF'
REL 'ON', MOG 'OFF'
REL 'ON', MOG 'ON'

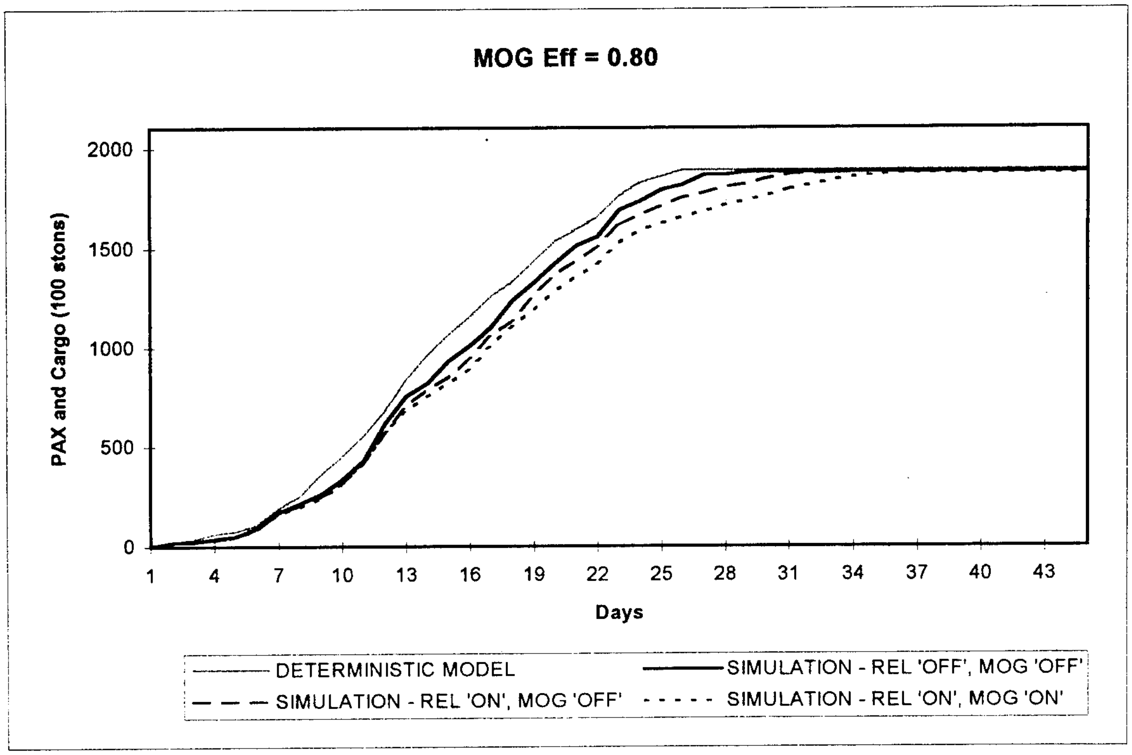

MOG Eff = 1.00
CDF - Landings
4000
3500
3000
2500
2000
1500
1000
500
0
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
Days
REL 'OFF', MOG 'OFF'
REL 'ON', MOG 'OFF'
REL 'ON', MOG 'ON'

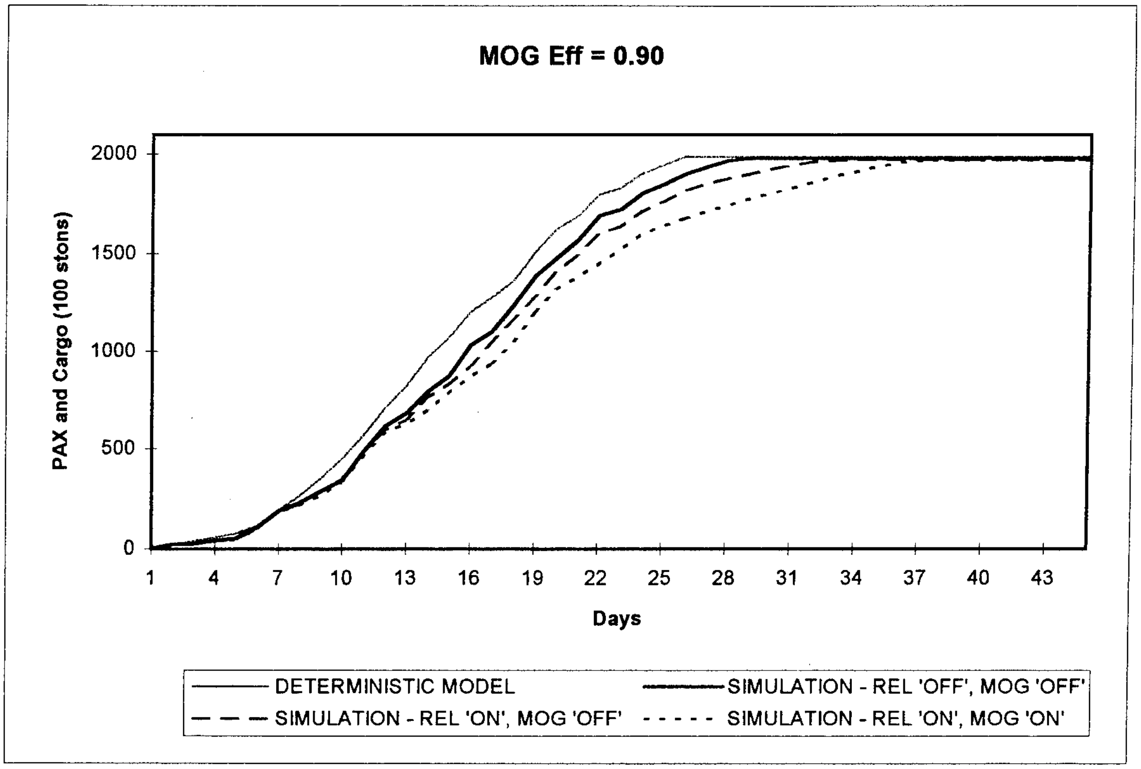
APPENDIX F. MOG EFFICIENCY FACTOR - STONS

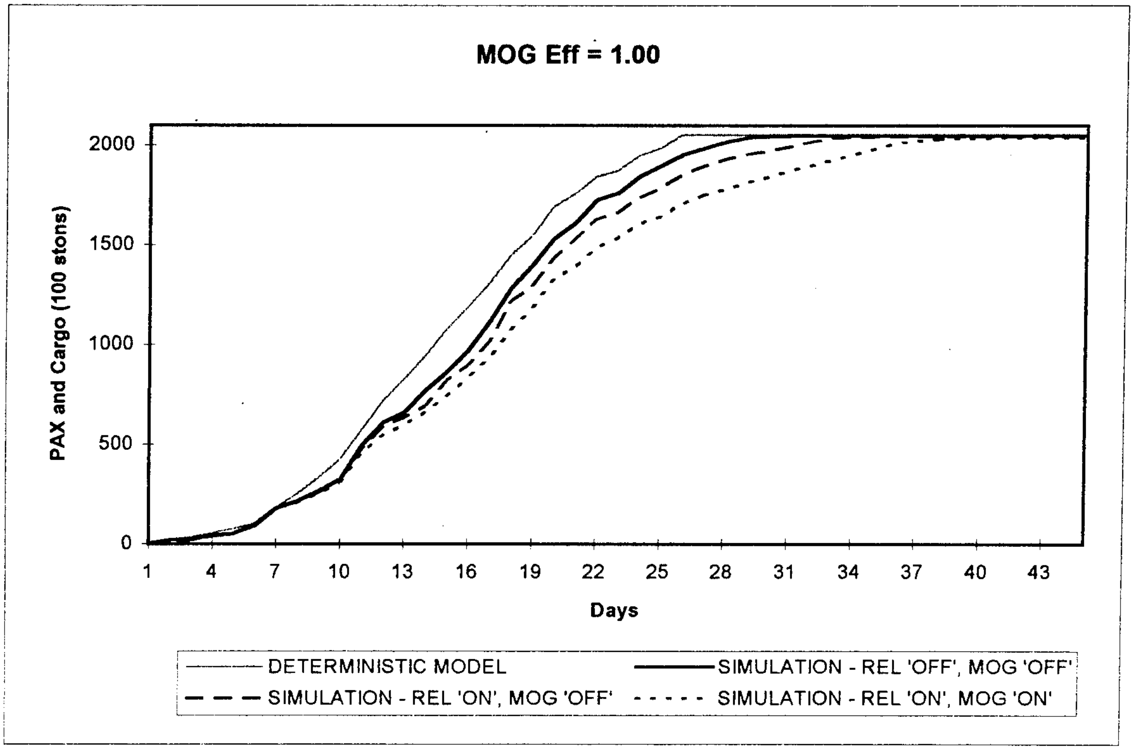

MOG Eff = 0.50


MOG Eff = 0.60


MOG Eff = 0.70
PAX and Cargo (100 stons)
2000
1500
1000
500
0
1
4
7
10
13
16
19
22
25
28
31
34
37
40
43
Days
DETERMINISTIC MODEL
SIMULATION - REL 'OFF', MOG 'OFF'
SIMULATION - REL 'ON', MOG 'OFF'
SIMULATION - REL 'ON', MOG 'ON'


MOG Eff = 0.80
PAX and Cargo (100 stons)
2000
1500
1000
500
0
1
4
7
10
13
16
19
22
25
28
31
34
37
40
43
Days
DETERMINISTIC MODEL
SIMULATION - REL 'OFF', MOG 'OFF'
SIMULATION - REL 'ON', MOG 'OFF'
SIMULATION - REL 'ON', MOG 'ON'


MOG Eff = 0.90
PAX and Cargo (100 stons)
0
500
1000
1500
2000
1
4
7
10
13
16
19
22
25
28
31
34
37
40
43
Days
DETERMINISTIC MODEL
SIMULATION - REL 'OFF', MOG 'OFF'
SIMULATION - REL 'ON', MOG 'OFF'
SIMULATION - REL 'ON', MOG 'ON'


MOG Eff = 1.00
PAX and Cargo (100 stons)
0
500
1000
1500
2000
1
4
7
10
13
16
19
22
25
28
31
34
37
40
43
Days
DETERMINISTIC MODEL
SIMULATION - REL 'OFF', MOG 'OFF'
SIMULATION - REL 'ON', MOG 'OFF'
SIMULATION - REL 'ON', MOG 'ON'

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7.	USAF Studies & Analyses Agency (Attn: CPT David Horton) 1570 Airforce Pentagon Washington, DC 20330-1570	3
8.	LT David Goggins Submarine School SOAC 96010 Box 700 Groton, CT 06349-5700	2